Logical Inference and Its Dynamics[1]

Carlotta Pavese

18 July 2016

[1]

Carroll[2]'s regress

[2] Lewis Carroll. What The Tortoise Said to Achilles. *Mind*, 4(14):278–80, 1895

- Start with premises A and *if* A *then* B:

  1. A
  2. *If* A *then* B

- How does one get from these premises to the conclusion B?
- Presumably, by appeal to *modus ponens*. But now, suppose the rule of *modus ponens* were identical to the universal statement **LT-mp**.[3]

[3] **LT-mp**: *For every* X, Y, *if* X *and* if X then Y, *then* Y.

- Then, presumably, following that rule would be a matter of instantiating this statement for the particular case of A and B.
- But by instantiating such a universal, one only gets to (3), still short of the conclusion B:

  1. A
  2. *If* A *then* B
  3. *If* A *and* if A then B *then* B

- How can one get from (1-3) to the conclusion? Again, by appeal to *modus ponens*, one would guess.
- The problem is that if following *modus ponens* is the same thing as using **LT-mp**, then arguing by *modus ponens* must amount to instantiating it for the particular cases of the premises.
- But by so doing, one will only get to the following four-premise argument (by taking the conjunction of A and *If* A *then* B as the first premise, and (3) as the second premise), and still short of the conclusion B.
- And so on. Therefore, if following *modus ponens* were the same as using a universal statement such as **LT-mp** in the course of an argument, following such a rule would trigger a regress, making it impossible to reach a conclusion.
- But we do routinely succeed at reasoning by *modus ponens*.
- So, the argument concludes that following *modus ponens* cannot be the same as using a universal statement such as **LT-mp**.

Diagnosis Following an inference rule is not the same as using a general principle in the course of an argument.

Now, from **Diagnosis**, it is a shot step to conclude to:

Rules versus General Principles An inference rule is not the same as a general principle.

Some proponents of Rules versus General Principles

Ryle ([4], p. 7): knowing a rule $\neq$ knowing a truth.
Rumfitt ([5], p. 358): knowledge of a general principle/truth $\neq$ our ability to make deductions.
Dummett ([6], p. 303): the moral of the regress is that an argument of the form

(A) *P. If P, Q. Therefore Q.*

cannot be identified with the conditional statement

(C) *If P, and If P then Q, then Q*

or with the universal sentence

(U) *For every P, Q, if P, and If P then Q, then Q*.

[4] George Ryle. Knowing How and Knowing That: the Presidential Address. *Proceedings of the Aristotelian Society, New Series*, 46:1–16, 1945

[5] Ian Rumfitt. Inference, Deduction, and Logic. In J. Bengson and M. Moffett, editors, *Knowing How: Essays on Knowledge, Mind, and Action*, pages 334–350. Oxford University Press, Oxford, 2011

[6] Michael AE Dummett. *Frege: Philosophy of Language*, volume 2. London: Duckworth, 1981

An Observation and A Question

1. *Rules versus General Principle* is only negative.
2. What is exactly the (semantic) distinction between (A) and (C)? Or between (A) and (U)?
3. More generally, what is the distinction between the argument:

   P/Q

   and the conditional statement:

   If P then Q.

Notes about 'Therefore'

1. Consider the inference $\ulcorner \phi_1, \phi_2, \phi_3$. *Therefore*, $\psi \urcorner$.
2. Grice [7]: a sentence such as "John is English and therefore brave" is truth-conditionally equivalent to "John is English and brave."
3. '*Therefore*' does not add to the truth conditions of the sequence $\ulcorner \phi_1, \phi_2, \phi_3, \psi \urcorner$.

[7] H Paul Grice. *Studies in the Way of Words*. Harvard University Press, 1991

A Dynamic Interpretation of Argument Schema

Notes about 'Therefore'

1. Consider $\ulcorner\phi_1, \phi_2, \phi_3$; *therefore*, $\psi\urcorner$.
2. The first three premises correspond to **update**: uttering them is an invitation to update the current context sequentially with ϕ_1, ϕ_2, and ϕ_3.
3. It is, moreover, plausible that the phrase $\ulcorner$*therefore*, $\psi\urcorner$ plays the role of the **test** part.
4. '*therefore*' is a deictic expression in that it refers back to the utterance of certain premises. Because of that, a dynamic interpretation of '*therefore*' — i.e., an interpretation that highlights the role played by the expression within a discourse — seems to be particularly fitting.
5. Thinking of '*therefore*' as a test captures the Gricean insight that in some sense, '*therefore*' is informationally empty.
6. A test is in a similar sense informationally empty: its utterance does not alter the context by eliminating assumptions.
7. If so, the overall meaning of a one-premise argument of the form $\ulcorner\phi$; *therefore* $\psi\urcorner$ can be thought of as a function that checks whether the context created by the utterance of the premise ϕ supports the conclusion ψ.

$$\textbf{test}\left\{\frac{\left.\phi_1, \ldots, \phi_n\right\}\ \textbf{update}}{\psi}\right.$$

Quote from van Benthem ([8], p. 11)

> The premises of an argument invite us to update our initial information state, and then, the resulting transition has to be checked to see whether it 'warrants' the conclusion (in some suitable sense).

[8] Johan van Benthem. *Exploring Logical Dynamics*. CSLI Publications and Stanford/Cambridge University Press, 1996

Following van Benthem, we can distinguish between *two* aspects of an inference and of an inference schema:

1. **update**: updating the initial set of assumptions;
2. **test**: checking whether the update has resulted in a set of assumptions that 'warrants', or supports, the conclusion.

Direct Argument for a Dynamic Conception of Inference Rules

Premise 1: The Dynamic Conception of Inference *An inference is a matter of moving from a set of assumptions to another set of assumptions which licenses the conclusion.*

Premise 2: Inference rules vis a vis inferences *Inference rules codify our inferences along certain structural dimensions.*

From 1, 2 *Inference rules are rules to move from a set of assumptions to another set of assumptions.*

Modeling Assumption *Rules to update sets of assumptions can be modeled as functions from sets of assumptions to sets of assumptions — i.e., as context-change potentials (*CCPs*)* (Heim[9], Kamp[10], [11], van Benthem[12], Veltman[13], and Groenendijk and Stokhof[14]).

Conclusion *Inference rules can be modeled as functions from sets of assumptions to sets of assumptions — i.e., as CCPs.*

[9] Irene Heim. The Semantics of Definite and Indefinite Noun Phrases. PHD Dissertation, 1982

[10] Hans Kamp. A Theory of Truth and Linguistic Representation. In *Formal Methods in the Study of Language (Groenendijk, J. A. G and M. Stokhof (eds.), pp. 277-321)*. Mathematish Centrum, Universiteit van Amsterdam, 1981

[11] Hans Kamp and Uwe Reyle. *From Discourse to Logic: Introduction to Model-theoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Number 42. Springer Science & Business Media, 1993

[12] Johan van Benthem. *Exploring Logical Dynamics*. CSLI Publications and Stanford/Cambridge University Press, 1996

[13] Frank Veltman. Defaults in Update Semantics. *Journal of philosophical logic*, 25(3):221–261, 1996

Towards the proposal

Definition 0.1. (**Dynamic Semantics**)

1. If σ has the form p, $c[\sigma] = \{w \in c\colon w \in \langle p \rangle\}$;
2. If σ has the form $\neg\phi$, $c[\sigma] = c - c[\phi]$;
3. If σ has the form ϕ & ψ, $c[\sigma] = c[\phi][\psi]$;
4. If σ has the form $\phi \vee \psi$, $c[\sigma] = c[\phi] \cup c[\neg\phi][\psi]$.

Definition 0.2. (Example of a Test)

If σ has the form *must*-ϕ, $c[\sigma] = \begin{cases} c & \text{if } c \vDash \phi \\ \emptyset & \text{if } c \nvDash \phi \end{cases}$

Definition 0.3. (Support) *c supports ψ* ($c \vDash \phi$) iff $c[\phi] = c$.

Definition 0.4. (Dynamic Entailment) $\phi_1, \ldots, \phi_n \vDash_{DE} \psi$ iff $\forall c$: $c[\phi_1] \ldots [\phi_n] \vDash \psi$.

Update (for &-E) $c[\phi_1$ & $\phi_2]$

$$\frac{\phi_1 \;\&\; \phi_2}{\phi_1, \phi_2}$$

Figure 1: &-Elimination

Test (for &-E) $c[/\phi_1, \phi_2] = \begin{cases} c & \text{if } c \vDash \phi_1 \quad \text{and} \quad c \vDash \phi_2 \\ \varnothing & \text{if either} \quad c \nvDash \phi_1 \quad \text{or} \quad c \nvDash \phi_2 \end{cases}$

Definition 0.5. (&-Elimination)

$$c([\phi_1 \& \phi_2] \circ [/\phi_1, \phi_2]) = \begin{cases} c[\phi_1 \ \& \ \phi_2] & \text{if } c[\phi_1 \ \& \ \phi_2] \vDash \phi_1, \phi_2 \\ \varnothing & \text{if } c[\phi_1 \ \& \ \phi_2] \nvDash \phi_1, \phi_2 \end{cases}$$

Update (for And-I) $c[\phi_1, \phi_2] = c[\phi_1][\phi_2]$

Test (for And-I) $c[/\phi_1 \ \& \ \phi_2] = \begin{cases} c & \text{if } c \vDash \phi_1 \ \& \ \phi_2 \\ \varnothing & \text{if } c \nvDash \phi_1 \ \& \ \phi_2 \end{cases}$

Definition 0.6. (&-Introduction)

$$c([\phi_1, \phi_2] \circ [/\phi_1 \ \& \ \phi_2]) = \begin{cases} c[\phi_1][\phi_2] & \text{if } c[\phi_1][\phi_2] \vDash \phi_1 \ \& \ \phi_2 \\ \varnothing & \text{if } c[\phi_1][\phi_2] \nvDash \phi_1 \ \& \ \phi_2 \end{cases}$$

$$\frac{\phi_1, \phi_2}{\phi_1 \ \& \ \phi_2} \ \textbf{\&-I}$$

Figure 2: &-Introduction

Update (for MP) $c[\phi, \phi \to \psi]) = c[\phi][\phi \to \psi]$

Definition 0.7. (Modus Ponens for the Material Conditional)

$$c([\phi, \phi \to \psi] \circ [/\psi]) = \begin{cases} c[\phi][\neg\phi \vee \psi] & \text{if } c[\phi][\neg\phi \vee \psi] \vDash \psi \\ \varnothing & \text{if } c[\phi][\neg\phi \vee \psi] \nvDash \psi \end{cases}$$

$$\frac{\phi \qquad \phi \to \psi}{\psi} \ \textbf{MP}$$

Figure 3: Modus Ponens

Definition 0.8. (Schema)

$$c([\phi_1, \ldots, \phi_n] \circ [/\psi]) = \begin{cases} c[\phi_1][\phi_2]\ldots[\phi_n] & \text{if } c[\phi_1, \ldots, \phi_n] \vDash \psi \\ \varnothing & \text{if } c[\phi_1, \ldots, \phi_n] \nvDash \psi \end{cases}$$

$$\frac{\phi_1, \ldots, \phi_n}{\psi}$$

Figure 4: Schema

The Case of Meta-rules

$$\frac{\begin{matrix} [\phi] \\ \vdots \\ \psi \end{matrix}}{(\phi \to \psi)} \to I$$

Figure 5: Conditional Proof

Definition 0.9. (Update for sub-proofs, first try)
If σ has the form $\phi_1, \ldots, \phi_n/\psi$, $c[\sigma] = c[(\phi_1 \ \& \ \ldots \ \& \ \phi_2) \to \psi]$.

Definition 0.10. (Update for sub-proofs)
If σ has the form $\phi_1, \ldots, \phi_n/\psi$, $c[\sigma] = \{w \in c \colon c[\phi_1 \ \& \ \ldots \ \& \ \phi_n] \vDash \psi\}$.

Definition 0.11. (Conditional Proof)

$$c([\phi/\psi] \circ [/\phi \to \psi]) = \begin{cases} c[\phi/\psi] & \text{if } c[\phi/\psi] \vDash \phi \to \psi \\ \varnothing & \text{if } c[\phi/\psi] \nvDash \phi \to \psi \end{cases}$$

$$\frac{\phi_1 \vee \phi_2 \qquad \begin{matrix} [\phi_1] \\ \vdots \\ \psi \end{matrix} \qquad \begin{matrix} [\phi_2] \\ \vdots \\ \psi \end{matrix}}{\psi}$$

Figure 6: Argument by Cases

Definition 0.12. (**Argument by Cases**)

$$c([\phi_1 \vee \phi_2, \phi_1/\psi, \phi_2/\psi] \circ [/\psi]) = \begin{cases} c* & \text{if } c* \vDash \psi \\ \emptyset & \text{if } c* \nvDash \psi \end{cases}$$

where c* = $c[\phi_1 \vee \phi_2][\phi_1/\psi][\phi_2/\psi]$

$$\frac{P_1, \ldots, P_n}{C}$$

Figure 7: Schema General

Definition 0.13. (**Schema-General**)

$$c([P_1, \ldots, P_n] \circ [/C]) = \begin{cases} c[P_1][P_2]\ldots[P_n] & \text{if } c[P_1, \ldots, P_n] \vDash C \\ \emptyset & \text{if } c[P_1, \ldots, P_n] \nvDash C \end{cases}$$

Validities of Rules

Definition 0.14. $P_1, \ldots, P_n \vDash_0 C$ iff for every c: $c([P_1, \ldots, P_n] \circ [/C]) \neq \emptyset$.

Theorem 0.1. $P_1, \ldots, P_n \vDash_0 C$ iff $P_1, \ldots, P_n \vDash_{DE} C$.

Proof. (Left to right) Suppose $P_1, \ldots, P_n \vDash_0 C$. Then, by **Definition 0.13**, for every c: $c([P_1, \ldots, P_n] \circ [/C]) \neq \emptyset$. But then, by **Definition 0.12**, for every c: $c[P_1, \ldots, P_n] \vDash C$. Hence, by **Definition 0.4**, $P_1, \ldots, P_n \vDash_{DE} C$. (Right to left) Suppose $P_1, \ldots, P_n \vDash_{DE} C$. Then, by **Definition 0.4**, for every c: $c[P_1, \ldots, P_n] \vDash C$. Then, by **Definition 0.12**, for every c: $c[P_1, \ldots, P_n] \circ c[/C] \neq \emptyset$. Then, by **Definition 0.13**, $P_1, \ldots, P_n \vDash_0 C$. □

Objection

- A possible objection is that this notion of validity ($\vDash_0$) conflates two notions of validity: *classical validity* and *dynamic validity*.
- As observed by van Benthem ([15], p. 11 and pp. 18-19), these are indeed different notions of validity. For dynamic validity, the order of the premises and the multiplicity of their occurrence matters.
- That seems to clash with the basic structural rules of standard classical logic.

[15] Johan van Benthem. *Exploring Logical Dynamics*. CSLI Publications and Stanford/Cambridge University Press, 1996

Response

- Can this important distinction between dynamic validity and classical validity be vindicated on the present approach? Classical entailment can be thought of as a special case of dynamic entailment — i.e., as dynamic entailment in *contexts of perfect information*.[16]
- Contexts of perfect information only include *the world of the context* — no other world is compatible with a set of propositions that

[16] .
William B Starr. A Uniform Theory of Conditionals. *Journal of Philosophical Logic*, 43(6):1019–1064, 2014a

completely distinguishes the actual world from any other possible worlds. So, let the context of perfect information relative to w be $\{w\}$. Classical entailment ($\vDash_{CE}$) emerges by focusing on perfect information:

Definition 0.15. $P_1, \ldots, P_n \vDash_{CE} C$ iff $\forall\{w\}$: $\{w\}[P_1] \ldots [P_n] \vDash C$.

Call a function from contexts of perfect information to other contexts of perfect information a *limiting dynamic semantic value*.

The limiting dynamic semantic value of a schema is insensitive to the order of its premises.

So when we want to highlight the insensitivity of classical structural rules to the order of their premises, we can then think of them as limiting dynamic semantic values.

This move preserves van Benthem's distinction, while clinging to the idea according to which classical inference rules are sorts of dynamic semantic values.

The Dynamic Conception of Inference Rules and Carroll's regress

1. The dynamic notion of inference rules offers a suitable and independently motivated semantic replacement of the notion of rules as logical truths.

    (a) For suppose inference rules are not logical truths but the sort of context-change potentials that I described.

    (b) Following an inference rule in this sense is not a matter of adding one more premise to the existing premises.

    (c) Rather, it is a matter of *implementing a particular function* — one that given the premises and a context as arguments, it simply outputs a value — it updates the context with the existing premises, having checked that the result supports the conclusion.

    (d) So the implementation of this particular function does not require adding any further premise to the context.

2. *If* one can follow a rule at all, one *could not* get stuck in the regress of the premises. Whereas the notion of following a logical truth is regress-triggering and hence paradoxical, the notion of following a rule developed here is not.

3. It captures the distinction between an argument P/Q and a conditional statement *If P then Q*.

4. (Modulo a suitable theory of contexts), it also captures the distinction between **Argument Schema** and **LT-mp**.[17]
5. (Modulo a suitable theory of contexts) It capture sthe distinction between **Argument Schema** and **Conditional Schema**.[18]
6. In the dynamic setting, we can also distinguish between **Argument Schema** and **LT-mp-Dynamic**.[19]

Primitive Recursive Functions and Inferential Competences

1. The dynamic notion of following a rule is non-paradoxical.
2. A different question: how to explain our ability to follow a rule?
   (a) On the dynamic conception of inference rules, being competent with an inference rule is a matter of *being competent with a function*.
   (b) What does it mean to be competent with a function?
   (c) Consider: *the biological father of*.
   (d) I may represent the function (grasp it) without being able to implement it.
   (e) Being competent with a function plausibly requires representing the function *primitively recursively*: representing it in terms of operations that the subject can already perform.
   (f) So if inference rules are dynamically conceived, our being competent with inference rules requires these functions being *primitive recursive for us* — for individual of average linguistic competence.
   (g) How plausible is this claim?

A sketch of a response

As shown by the semantic clauses in **Dynamic Semantics**, *all* the relevant dynamic semantic values are defined through composition, recursion, and set theory in terms of a basic update function — **atoms**:

atoms If $\sigma = p$: $c[\sigma] = \{w \in c : w \in \langle p \rangle\}$

The composition of **update** and **test** will be itself a primitive recursive function (for us), provided that **atoms** and **test** are themselves primitive recursive functions.

[17] **LT-mp** *For every* X, Y, *if* X *and* if X then Y, *then* Y.
Argument Schema *X*, *if X then Y* / *Y*.

[18] **Conditional Schema**: If *X* and *if X then Y*, then *Y*

[19] **LT-mp-dynamic** *For every* X, Y, *if* X *and* if X then Y, *then must*-Y

This explanation does depend on atoms and **test** being primitive recursive functions. However, note that accepting that they are simply boils down to accepting that humans are capable of recognizing inconsistency relations between atomic sentences. And this does seem to be a capacity for acquiring any sort of linguistic competence.

Moreover, their being primitive recursive functions leaves open that both **atoms** and **test** may be definable in terms of more basic capacities that humans possess, rather than being themselves basic. For example, consider **test**. A more complete explanation will define it in terms of even more basic test functions. For example, it will define it in terms of functions testing the relation of supports between contexts and *atomic sentences* first; then, it will define a test for checking the relations of support between context and *non-atomic sentences*; finally, it will reduce both to relations of inclusion between propositions. In the case of **atoms**, instead, a more detailed explanation will have to define **atoms** in terms of the dynamic semantic values of sub-sentential expressions, which is customary to do, when one gives a definition of dynamic semantic values for a wider fragment of English.

References

Lewis Carroll. What The Tortoise Said to Achilles. *Mind*, 4(14): 278–80, 1895.

Michael AE Dummett. *Frege: Philosophy of Language*, volume 2. London: Duckworth, 1981.

H Paul Grice. *Studies in the Way of Words*. Harvard University Press, 1991.

Jeroen Antonius Gerardus Groenendijk and Martin Stokhof. Dynamic Predicate Logic. *Linguistics and Philosophy*, pages 39–100, 1991.

Irene Heim. The Semantics of Definite and Indefinite Noun Phrases. PHD Dissertation, 1982.

Hans Kamp. A Theory of Truth and Linguistic Representation. In *Formal Methods in the Study of Language (Groenendijk, J. A. G and M. Stokhof (eds.), pp. 277-321)*. Mathematish Centrum, Universiteit van Amsterdam, 1981.

Hans Kamp and Uwe Reyle. *From Discourse to Logic: Introduction to Model-theoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Number 42. Springer Science & Business Media, 1993.

Ian Rumfitt. Inference, Deduction, and Logic. In J. Bengson and M. Moffett, editors, *Knowing How: Essays on Knowledge, Mind, and Action*, pages 334–350. Oxford University Press, Oxford, 2011.

George Ryle. Knowing How and Knowing That: the Presidential Address. *Proceedings of the Aristotelian Society, New Series*, 46:1–16, 1945.

William B Starr. A Uniform Theory of Conditionals. *Journal of Philosophical Logic*, 43(6):1019–1064, 2014a.

Johan van Benthem. *Exploring Logical Dynamics*. CSLI Publications and Stanford/Cambridge University Press, 1996.

Frank Veltman. Defaults in Update Semantics. *Journal of philosophical logic*, 25(3):221–261, 1996.