Introduction to Bayesian Epistemology

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Outline

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- Beliefs are full beliefs: you either believe that something is the case, or not.
- In other words, beliefs are an all or nothing matter.

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- Credences differ from full beliefs in that they come in degree.
- Credences are degrees of confidence in certain claims.

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- Moreover, it is not immediately clear in what sense credences are true or false.
- Credences can be more or less accurate, but not (really) true or false.

Bayesian Epistemology

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- Bayesianism models degrees of belief as probabilities along the lines of Kolmogorov's (1933) axiomatization.

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- Saying that credences should be modeled as probabilities amounts to saying that credences satisfy the axioms of probability.

Probability Axioms

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- If A is a tautology, then $Pr(A)=1$;
- If A is a contradiction, then $Pr(A)=0$;
- If A and B are incompatible, then $Pr(AvB) = Pr(A) + Pr(B)$.

Outline

Conditional credences

- A credence in p can be unconditional or conditional on other propositions' being true.
- A conditional credence is a credence of the form: $Cr(p|q)$.
- Conditional credences can be defined in terms of unconditional ones.

Conditional Credences

Conditional Credences

$Cr(p|q) = Cr(p\&q)/Cr(q)$.

Provided that $Cr(q)>0$

Conditional Credences

- $Cr(p|q) = Cr(p)$, then p and q are independent relative to Cr.

Conditionalization

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- By itself, the definition of conditional probability is of little epistemological significance.
- It acquires epistemological significance only in conjunction with a further epistemological assumption:

Conditionalization

Simple Principle of Conditionalization

If one begins with initial or prior credence Cr_i, and one acquires new evidence which can be represented as becoming certain of an evidentiary statement E (assumed to state the totality of one's new evidence and to have initial probability greater than zero), then rationality requires that one systematically transform one's initial probabilities to generate final or posterior credences Cr_f by conditionalizing on E—that is: Where p is any statement, $Cr_f(p) = Cr_i(p|E)$.

Confirmation

Bayesianism offers a nice account of what it means for a piece of evidence to confirm one's theory:

Confirmation

e confirms h (relative to Cr) iff $Cr(h|e) > Cr(h)$.

Bayesianism

- Bayesianism can be understood as combining a synchronic thesis about the degrees of belief or credences of a rational agent at a given time,
- But also as a diachronic thesis about how they evolve in response to evidence.
- Synchronically, the agent's credences are probabilities.
- Diachronically, her credences update according to the rule of conditionalization.

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By now, we should be able to appreciate the differences between Bayesian Epistemology and Traditional Epistemology:

- Knowledge and beliefs are binary notions—an all or nothing matter.
- On the other hand, credences come in potentially infinitely many degrees.
- Knowledge involves objective notions—such as safety and truth.
- It is not clear whether there is anything corresponding to safety and truth within the Bayesian framework.

Arguments for Bayesian Epistemology

Why think that Bayesian Epistemology, then, provides a better framework within which to understand epistemological notions?

Arguments for Bayesian Epistemology

There are several reasons. Let us look at some of them.

Decision Theory

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- On the other hand, traditional epistemology offers no decision theory.

Observation and evidence

Hajek: "Observations rarely deliver certainties—rather, their effect is typically to raise our probabilities for certain propositions (and to drop our probabilities for others), without any reaching the extremes of 1 or 0. Traditional epistemology apparently has no way of accommodating such less than conclusive experiential inputs, whereas Jeffrey conditionalization is tailormade to do so."

Its Foundations

"Bayesianism has powerful mathematical underpinnings. It can help itself to a century of work in probability theory and statistics. Traditional epistemology may appeal to the occasional system of epistemic or doxastic logic, but nothing comparable to the formidable formal machinery that we find in the Bayesian's tool kit."

Applications

"Bayesian method", in turn, have much wider application than any formal systematization of "knowledge" or "belief". Look at the sciences, social sciences, engineering, and artificial intelligence if you need any convincing of this."

Introduction to the Principal Principle

- There seem to be at least two different concepts of probability: the probability that is involved in degrees of belief (epistemic or subjective probability) and the probability that is involved in random events, such as the tossing of a coin (chance).

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- Some people think that this was a mistake and that there was only one kind of probability, subjective probability.
- For Bayesians who believe in both kinds of probability, an important question is: What is (or should be) the relation between them?

Outline

The Principal Principle

Lewis (1980)'s Principal Principle says that one's credence Cr in a proposition p given that one is certain the chance Ch of p's being true is x must be identical to x:

Principal Principle

$Cr(p|Ch(p)=x) = x$, if $\mathrm{Cr}(Ch(p)=x)=1$.

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- The Principal Principle bridges the gap between credences and subjective probability (=degrees of confidence) on one hand and objective chances and likelihoods on the other.
- As Hajek puts it, "The idea is that one should align one's credences with what one takes the corresponding objective chances to be, where the latter are genuine probabilities in the world."

Objections to the Principal Principle

Crystal Balls

Suppose that there are crystal balls—devices which gave infallible predictions with respect to the outcomes of future events. And suppose that a crystal ball tells us at t_0 that the outcome of some chance event at t_1 will be A. Finally, suppose that the chance at t_0 of this outcome, A, is $1/2$. If our total evidence consists of the chance theory at this world and the history up to t_0, TH, what should our credence in A be? The correct answer seems to be 1. But PP2 appears to give the wrong answer—i.e. $1/2$.

The Reflection Principle

van Fraassen's Reflection Principle

If $Cr_t(Cr_{t'}(p) = x)$=1, $Cr_t(p|Cr_{t'}(p) = x) = x$ (for all t, t', a and x for which this is defined).

Here Cr_t is one's credence at time t, and $Cr_{t'}$ one's credence at later time t' .

The Reflection principle

The idea is that rationality requires a certain commitment to one's future opinions; when all is going well, one's future selves are better informed versions of one's current self.

The Reflection Principle

Briggs 2009

To understand what this means, it will help to imagine agents who satisfy Reflection. Such agents treat their future selves as experts about all propositions. An agent who satisfies Reflection and is certain that his or her future self believes A to degree r, believes A to degree r.

Counterexamples to van Fraassen's Reflection Principle

Future irrationality (David Christensen (1991))

The drug LSQ makes people believe to degree .99 that they can fly by flapping their arms. At t_0, you become certain that you will take LSQ before t_1. You deduce that at t_1, you will place credence .99 in the proposition (call it F) that you can fly. Thus, $Cr_0(Cr_1(\text{F}) = .99) = 1$. By Reflection, $Cr_0(\text{F})$ should be .99. This is clearly the wrong advice; your taking LSQ is not evidence that you can fly.

Memory Loss

William Talbott (1991)

At t_0, you are eating a dinner of spaghetti and meatballs. You expect to forget this by t_1, but you'll remember that t_0 was your dinner time. You'll also remember that you eat spaghetti for dinner 10 percent of the time. Where S is the proposition that you eat spaghetti at t_0, $Cr_0(Cr_1(\mathrm{S}) = .10) = 1$. Reflection advises you to set $Cr_0(\mathrm{S})$ equal to .10. But $Cr_0(\mathrm{S})$ should be much higher at t_0, your senses report the presence of spaghetti, and you should trust your senses.

Apparent memory loss

Arntzenius (2003)

A group of monks has elected to escort you to the city of Shangrila. The monks choose the route based on the outcome of a fair coin flip. If the coin lands heads, you will travel by the mountains; if tails, by the sea. If you travel by the mountains, you will arrive at Shangrila with glorious memories of the mountains. If you travel by the sea, your memories of the sea will be removed and replaced with glorious memories of the mountains. At t_0, you find yourself on the mountain path with the monks. You recognize that at t_1, after you've arrived, you will place credence $1/2$ in the proposition that you traveled by the mountains. Thus, where M is the proposition that you travel by the mountains, Reflection advises you to set $Cr_0(\text{M} \mid Cr_1(\text{M}) = .5)$ equal to .5. But $Cr_0(\text{M})$ should be 1.

Future misleading evidence

Maher (1992)

You are 90 percent certain that your friend Persi, a magician, knows the outcome of a fair coin toss. You also know that Persi is preternaturally eloquent and can persuade you to grant credence 1 to the proposition that he knows the outcome of the coin toss. Where H is the proposition that the coin lands heads, Reflection demands that you set $Cr_0(H|Cr_1(H) = 1)$ equal to 1. This is bad advice. Right now, you surely know better than to place so much trust in Persi's testimony!