# The Place for Knowledge in Bayesian Epistemology

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# Outline

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- According to Bayesian Epistemology, credences should be modeled as probabilities.
- Today, we will look at one important motivation for thinking of credences as probabilities—i.e., the Dutch Book Arguments.

# Dutch book arguments

- A Dutch book argument is an argument to the effect that a subject whose credences do not satisfy the probability axioms will be vulnerable to losing were the subject to engage in bets.

# The form of Dutch book arguments

1. If Cr(A) = q, then the agent's credences condone buying or selling, for an arbitrary sum of money $Sq$, a ticket which entitles the buyer to $S$ out of the seller?s pocket if A is true, and nothing otherwise.

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2. If Cr violates purported norm $N$, then the agent's credences condone entering into a Dutch book—that is, a set of bets which ensure that she suffers a net financial loss.
3. If an agent's credences condone entering into a Dutch book, then his or her credence function is incoherent.

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  Any agent who violates N has an incoherent credence function.
- The Dutch Book Theorem is supposed to provide evidence for premise (2) of a Dutch Book Argument.

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- With de Finetti, it is here assumed that a bet on a proposition H is an arrangement that has the following canonical form:

# Bet table

Table: **Bet table**

| H | Payoff |
|---|---|
| **True** | $S - qS$ |
| **False** | $-qS$ |

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- where $S$ is won if $H$ is true.
- $S$ is called the stake, as it is the total amount involved in the wager, that is the payoff in the case that $H$ is true together with the amount forfeited if $H$ is false.
- The quantity $q$ is called the betting quotient, which is the amount lost if $H$ is false $(=qS)$ divided by the stake $S$.

# Example

Table: **Horses table**

| Horse | Offered Odds | Implied Probability | Bet Price |
|---|---|---|---|
| **1** | even | 0.5 | 100 dollars |
| **2** | 3 to 1 against | 0.25 | 50 dollars |
| **3** | 4 to 1 against | 0.2 | 40 dollars |
| **4** | 9 to 1 against | 0.1 | 20 dollars |

# Example

Table: **Horses table**

| Bet Price | Bookie pays if Horse wins |
|---|---|
| 100 dollars | 100 dollars stake + 100 dollars |
| 50 dollars | 50 dollars stake + 150 dollars |
| 40 dollars | 40 dollars stake + 160 dollars |
| 20 dollars | 20 dollars stake + 180 dollars |

# Bet table

Table: **Bet table**

| Horse 1 winning | Payoff |
|---|---|
| True | $200-100$ |
| False | $-100$ |

# Bet table

Table: **Bet table**

| Horse 2 winning | Payoff |
|---|---|
| True | x-y |
| False | -y |

# Bet table

Table: **Bet table**

| Horse 3 winning | Payoff |
| --- | --- |
| True | x-y |
| False | -y |

# Bet table

Table: **Bet table**

| Horse 4 winning | Payoff |
|---|---|
| True | x-y |
| False | -y |

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- It is easy to show how it is possible to make book against someone with betting quotients that violate the probability axioms.
- Let $Q(H)$ be the agent's betting quotient for $H$. Assuming that the agent's betting quotients violate the axioms, a bookie can guarantee himself a profit by placing bets with the agent as described below.
- For simplicity the stake is set here at 1, but the following recipes for constructing a book against such a person are easily adapted for other stakes.

# Axiom 1

- Suppose $Q(H)$ is negative—i.e., less than 0.

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- In this case, the bookie buys the bet that pays 1 if $H$ is true and 0 otherwise, for the negative price $Q(H)$, which means that the agent collects $Q(H)$, and pays out 1 if $H$ is true, and 0 otherwise. Here the agent is betting against $H$ and the payoff table for the agent is as follows:

# Axiom 1

Table: **Bet table**

| H | Payoff |
|---|---|
| True | $-[1\text{-}Q(H)]$ |
| False | $Q(H)$ |

# Axiom 1

- Since $Q(H)$ is negative, the agent will suffer a net loss whatever the truth value of $H$.

# Axiom 2

- Suppose that an agent's betting quotient for a tautology (or a logical or necessary truth) $H$ is not equal to 1.

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- Suppose that an agent's betting quotient for a tautology (or a logical or necessary truth) $H$ is not equal to 1.
- The case where $Q(H)>1$ was included above, so assume that $Q(H)<1$.
- Here the bookie will buy the bet in which the agent pays the bookie 1 if $H$ is true, and nothing if $H$ is false, for $Q(H)$. The payoff table for the agent will be:

# Axiom 2

Table: **Bet table**

| H | Payoff |
|---|---|
| True | $-$[1-Q(H)] |
| False | $Q(H)$ |

Notice that since $H$ is a tautology (or logical or necessary truth) it must be true, which means that at the conclusion of the bet, the agent will have lost $[1-Q(H)]$.

# Axiom 3 Additivity

Assume that $H1$ and $H2$ are mutually exclusive and that $Q(H1vH2) \neq Q(H1) + Q(H2)$. There are two cases,

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- $Q(H1vH2) < Q(H1)+Q(H2)$.

# Axiom 3

## Additivity

If $Q(H1vH2) < Q(H1) + Q(H2)$, then the bookie will offer the agent the bet that pays 1 dollars if $H1$ and 0 otherwise for $Q(H1)$ and the bet that pays 1 if $H2$ is true and 0 otherwise for $Q(H2)$. The bookie then buys the bet that will pay him 1 dollars, if $(H1vH2)$ is true and 0 otherwise, for the price of $Q(H1vH2)$. The possible payoffs to the agent are summed up in the following table:

# Axiom 3

Table: **Bet table**

| H1 | H2 | Net Payoff |
|---|---|---|
| True | False | $[1-Q(H1)-Q(H2)+Q(H1vH2)-1]$ |
| False | True | $[1-Q(H1)-Q(H2)+Q(H1vH2)-1]$ |
| False | False | $[-Q(H1)-Q(H2)+Q(H1vH2)]$ |

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- Since $Q(H1 v H2) < Q(H1) + Q(H2)$, the agent loses in each case and thus the collection of bets assures a loss.

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- If $Q(H1vH2) > Q(H1) + Q(H2)$, then the bookie simply reverses the direction of the bets.
- Letting $V(H)$ be the payoff if $H$ is true, the expected value of a bet on $H$ is expressed by the equation: $\text{Exp}(H) = V(H)Q(H) + V(-H)(1 - Q(H))$.

# Conclusion of the Dutch Book Argument

- For each axiom, the individual bets involved in making the book are fair, which is to say they have an expected value of zero, when calculated using the agent's betting quotients, yet collectively they will produce a sure loss.
- The Dutch Book argument assumes that an agent's degrees of belief are linked with her betting quotients.
- This together with the theorem establishes that degrees of belief that violate the probability axioms are associated with bets that are fair in the above sense, but that lead to a sure loss.

# Conclusion of the Dutch Book Argument

- The argument then concludes that an agent's credences ought to obey the axioms of probability.
- Hence, they must be probabilities themselves.

# Outline

Why Probabilism?

Arguments against Bayesian Epistemology

A place for knowledge in Bayesian Epistemology

# The Problem of Old Evidence

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# The Problem of Old Evidence

- The Bayesian view of confirmation is not beyond problems, however.
- If $Cr(e) = 1$, then $e$ apparently cannot confirm anything by Bayesian lights.
- In that case, $Cr(h|e) = Cr(H \cap e)|Cr(e) = Cr(h)$.
- Yet we often think that such "old evidence" can be confirmatory.

# The Problem of Old Evidence

*Consider the evidence of the advance of the perihelion of Mercury, which was known to Einstein at the time that he formulated general relativity theory, and thus (we may assume) was assigned probability 1 by him. Nonetheless, he rightly regarded this evidence as strongly confirmatory of general relativity theory. The challenge for Bayesians is to account for this (Hajek p. 13).*

# Outline

# Synthesis?

*Should we really prefer one approach to epistemology over the other? Should one of the two approaches be jettisoned?(Hajek p. 17).*

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- Arguably, however, knowledge is a concept that plays an important explanatory role in our conceptual scheme.
- For example, we talk about what we know all the time. By contrast, we rarely (if ever) talk about credences. (Although we do talk about degrees of confidence that those credences formalize).

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- Common sense gives an important role to knowledge.
- Is it possible to reconcile common sense with Bayesian epistemology?

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It seems that there are several possible answers to this question:

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**Skepticism about knowledge** Bayesianism does not have a room for knowledge.

**Compatibilism** It is possible to find a place for knowledge within Bayesianism.

# Compatibilism

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**The Threshold View** Weatherson (2005)

**Knowledge as probability 1 on one's evidence** Williamson (2000).

**Knowledge as arbitrarily low credence** Moss (2014).

# The threshold view

## Weatherson 2005

It is tempting to say that $S$ believes that $p$ iff $S$'s credence in $p$ is greater than some salient number $r$, where $r$ is made salient either by the context of belief ascription, or the context that $S$ is in.

# Problems with the Threshold view

## The Arbitrariness objection (Weatherson 2005)

... any number $r$ is bound to seem arbitrary. Unless these numbers are made salient by the environment, there is no special difference between believing p to degree 0.9786 and believing it to degree 0.9875. But if $r$ is 0.98755, this will be the difference between believing $p$ and not believing it, which is an important difference.

# Possible responses to the problems

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- Perhaps it might be vague what is the correct number $r$?

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- Another option is to equate knowledge with credence 1 on one's evidence.
- But if credences are degrees of confidence, and knowledge is degree 1, then knowledge is the maximum degree of confidence, which is weird.
- So presumably this option requires rethink the idea that credences are degrees of confidence.

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- Yet another option is to think that knowledge is possibly any credence, of arbitrary low value.
- According to this vlew, any credence can be knowledge, if other requirements are satisfied.

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- This view leaves a lot to be explained.
- What does it mean for a credence to be true? Or safe?
- Without an answer to this question, it is hard to assess this view.