What is a Computer?

Programming, Memory and Limits of Computation

Carlotta Pavese

10.22.13

Outline

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Review

From Minds to Machines

- Dualism 👎, Identity Theory 👎, Functionalism 👍

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- So what exactly is computation?
- Could the things minds do amount to computation?

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- But neither do Finite State Machines
- And FSMs are another important building block of computers

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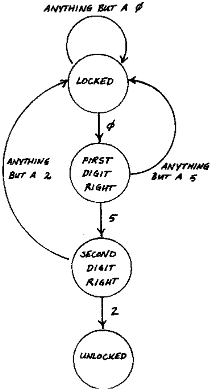
Finite State Machines

Combo Lock For 0-5-2 (p.36)



Finite State Machines

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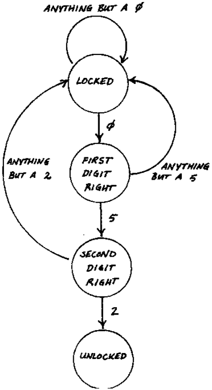
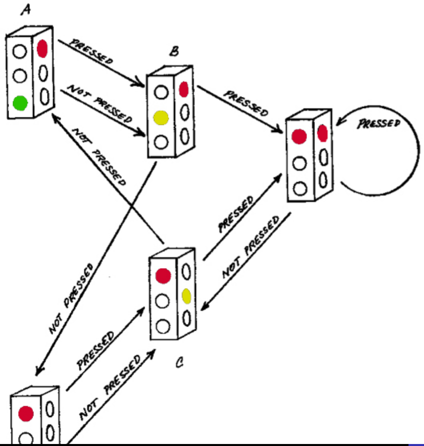


FSMs Listen to Themselves

- Response = input + current state
- Current state = previous input

Stoplight Controller

As a Finite State Machine



How A Stoplight Works

In Terms of Input/Output

Inputs:		*Outputs:*		
Walk Button	**Current State**	**Main Road**	**Cross Road**	**Next State**
Not Pressed	A	Red	Green	B
Not Pressed	B	Red	Yellow	D
Not Pressed	C	Yellow	Red	A
Not Pressed	D	Green	Red	C
Not Pressed	Walk	Walk	Walk	D
Pressed	A	Red	Green	B
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Finite State Machines

The Mealy Machine

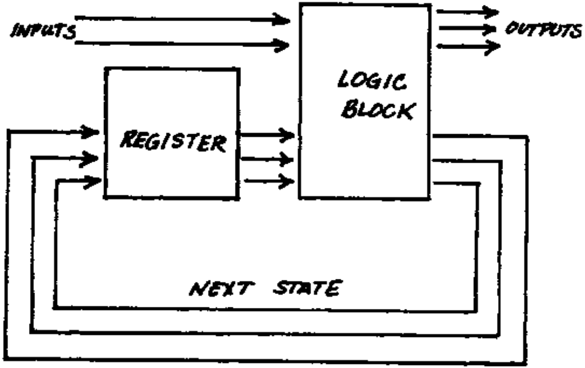

FIGURE 14

Finite State Machines

In General

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1. Register built from Boolean circuits

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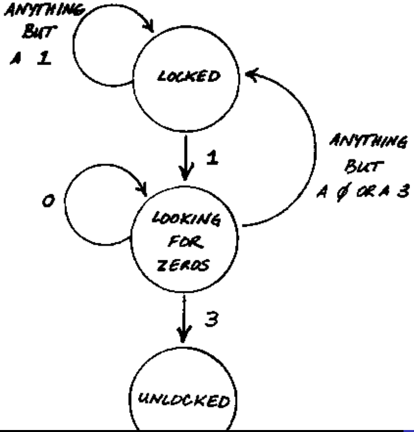
- All patterns?

Finite State Machines

Recognizing Patterns: begins w/1, any number of 0s, ends w/3 (p.35)



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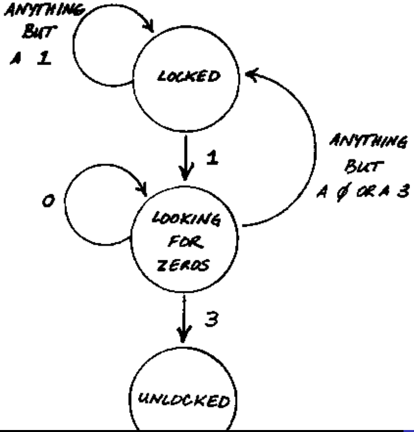

Important

There is one state for each digit that has to be remembered.

Minds Aren't Just Logic Blocks

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- Logic blocks always react to the same signal/input in the same way, but things with minds don't!

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- 'Register': a limited, finite memory
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- The register itself is a logic block: a configuration of bits that changes upon input

Finite State Machines

Recognizing Patterns

Palindrome

A string of symbols which is the same whether read backwards or forwards.

- E.g. 99, 757, 1001, abba, abccba, dad, mom

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- A finite state combo lock that detects only palindromes is not possible (even if alphabet is finite)

Finite State Machines

Recognizing Patterns

Why

To recognize a palindrome, you need to remember every digit of the first half. You need one state for each such memory. But there are infinitely many first halves, so you would need infinitely many states.

Finite State Machines

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- Isn't this a sentence: *I don't believe that* S?

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From Finite State Machines to Computers

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- Final ingredient of a computer: a more sophisticated kind of memory

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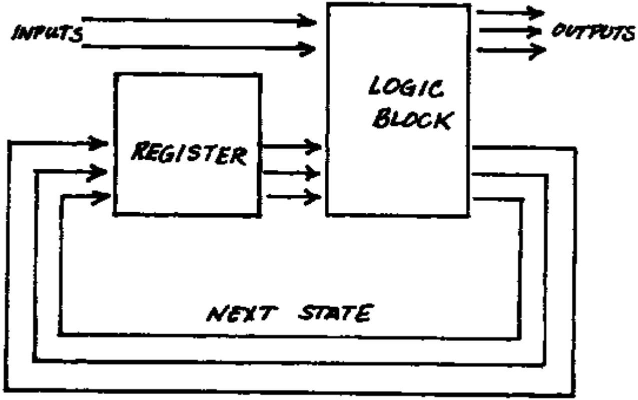
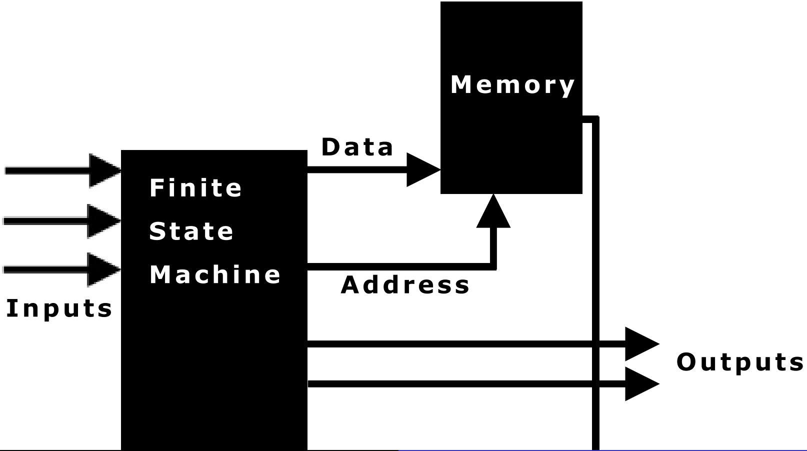


Finite State Machine

With a Real Memory



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Memory, Registers, Words, Addresses, Reading, Writing

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- Data are treated as input, instructions are executed

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- A logic block then adds these bit patterns, and writes the result to the memory

Outline

A General Purpose Computer

How Do You Get a Computer To Do Your Bidding?

Zuse Z3 (1941): 1st Programmable Computer

- How does a computer interpret bit patterns that are to be treated as instructions?
- How do you program one to do new things?

Machine Language

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  - Stored in special register: program counter
- Normally computer will just move from one register to next, but control instructions can tell it to skip some or repeat some until a condition is met (loop)

Programming Languages

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- The computer does this by translating the programming language into its machine language

For Example

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- So Logo is just a convenient Macro!

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8. Crunching bits is manipulating a physical substance to send one of two possible signals 1 or 0

Outline

Introduction

Finite State Machines

Computers

Programming

Computation and Turing Machines

Extents and Limits of Computation

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   - Halting Problem: no computer will ever solve it

Calculation

A Person, A Pencil and Some Paper

- You start by writing some things down

$$X - Z = Y$$

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- You follow some rules for using them to get some thing else
- You do this by writing out each application of the rule
- These rules have the form: if you've written X, you can write Y

$$X - Z = Y$$

Calculation and Computation

Turing's Staring Point

Effective Calculation

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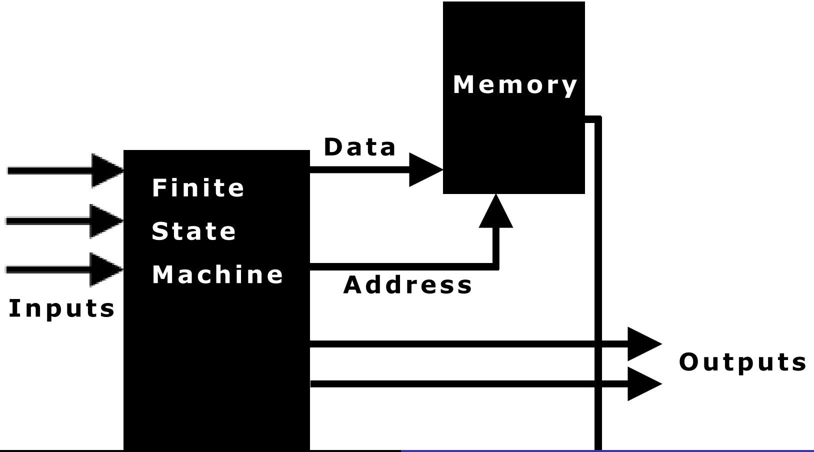
Turing: to compute is just to effectively calculate

A Computer

FSM with a Memory



Computation and Calculation

Turing Machines

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- 3 machine commands: write, move head left ($\ll$), move head right ($\gg$)

Turing Machines

And Programs

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Turing Machines

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Visualizing A Turing Machine

Executing This Program

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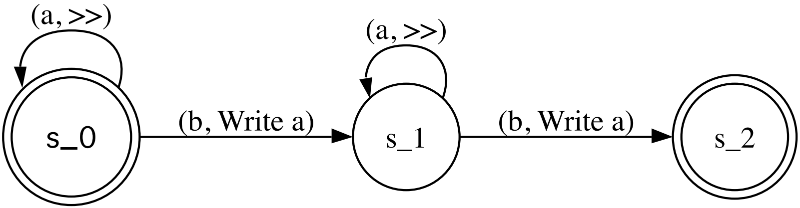

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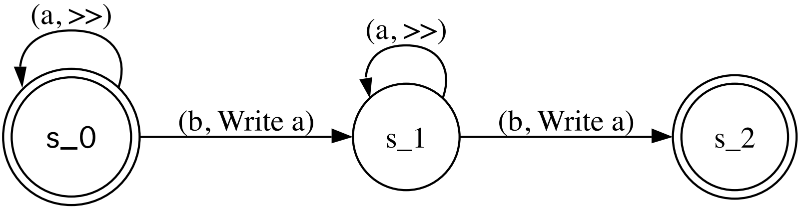

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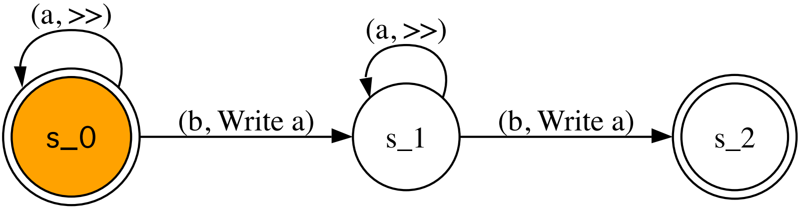

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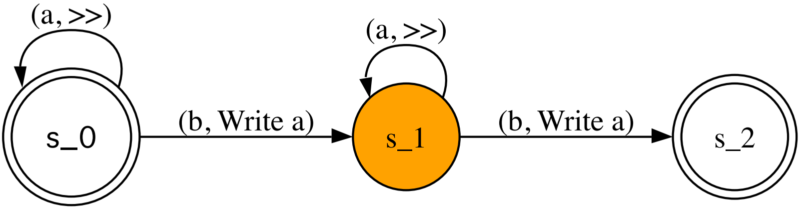

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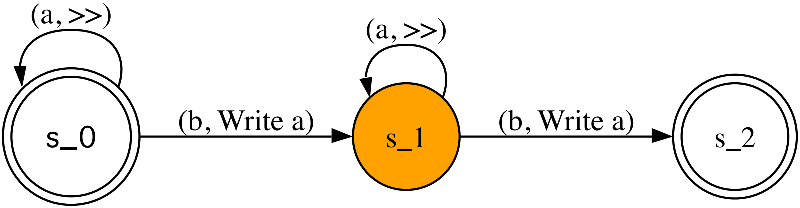

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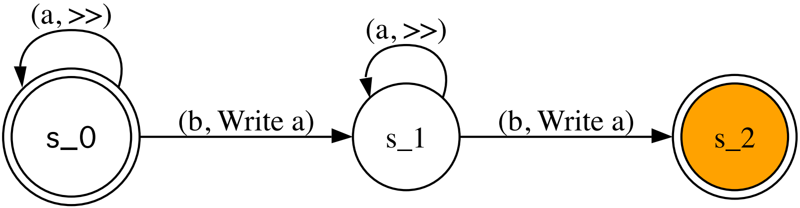

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About Programs and Turing Machines

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When a Turing Machine executes a program, what part of the machine is causing it to follow the routine? Where is the program in the machine and how does it cause the memory and states to change?

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- Recall that the Turing Machine consists of the tape assembly wired to a FSM
- A Turing Machine can be programmed by constructing the internal configuration and wiring of the FSM in a particular way
- But Turing showed there's another possibility

Programs and The Universal Turing Machine

The Programs Can Also be Stored on the Tape!

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Universal Turing Machine (UTM)

Given the appropriate tape, the UTM can do anything that any other Turing Machine can do. With the right software, it can do anything! (Though maybe much slower!)

The Church-Turing Thesis

Effective Calculation

The Church-Turing Thesis (Widely Accepted Conjecture)

Any effective calculation can be performed by a Turing Machine.

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Outline

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- Well, think back to our discussion of Rock-Paper-Scissors

Rock-Paper-Scissors

Not A Binary Input, Not A Binary Output!

Input A	Input B	Output
Scissors	Scissors	Tie
Scissors	Paper	A wins
Scissors	Rock	B wins
Paper	Scissors	B wins
Paper	Paper	Tie
Paper	Rock	A wins
Rock	Scissors	A wins
Rock	Paper	B wins
Rock	Rock	Tie

Rock-Paper-Scissors

Using Two Binary Inputs to Mimic One Ternary Input

Scissors = 01
Paper = 10
Rock = 11
A wins = 10
B wins = 01
Tie = 00

A Inputs	B Inputs	Outputs
01	01	00
01	10	10
01	11	01
10	01	01
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- So are they more powerful?

Analog Computation

Noise

- Instead of responding differently to some/no voltage, it will respond differently to any fraction of voltage differently

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  - The jury's still out on that one

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Something Computers Can't Do

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Is it possible to write a program that inspects any other programs and determines whether or not they will eventually stop?

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Answer: No, it's not possible. You can always build a

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- Programs are representations of algorithms in some programming language.
- There are usually several ways of expressing in algorithms, depending on the programming language.

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Answer: We can describe the speed of an algorithm on the basis of how much the time required for performing the task grows along with the size of the problem.

Algorithms

Comparing Algorithms

First sock-matching algorithm

- Pull out a random sock out of the basket.
- Pull out a second sock out of the basket.
- Compare it with the first pulled out.
- If it matches, bundle them together.
- if it does not match, throw it back in the basket and restart the process.

Algorithms

Comparing Algorithms

Second sock-matching algorithm

- Pull out a random sock out of the basket on the table.
- Pull out a second sock out of the basket.
- Compare it with the first pulled out.
- If it matches with any sock on the table, bundle them together.
- if it does not match, set it next to the first.
- Repeat the process until all the socks are matched.

Algorithms

Comparing Algorithms

Compare the first and the second algorithm

- Assume that there are n socks in the basket.
- Using the first algorithm, finding two that match requires pulling out and pulling back an average of half of the remaining socks.
- So the number of sock removals is proportional to the square of number of socks.
- The algorithm is of order n^2, meaning that for large problem, the time of execution grows as the square of the problem size.

Algorithms

Comparing Algorithms

Compare the first and the second algorithm

- In other words, if there are m times as many socks in the basket, the time of execution of the second algorithm will only by m by n.
- The time of execution of the first algorithm will be instead m^2 by n.
- So if n=10, then using the second algorithm to match twice as many socks will take twice as much time.
- Instead, if n=10, then using the first algorithm to match twice as many socks will take four times as much time.

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Comparing Algorithms

Compare the first and the second algorithm

- In other words, if there are m times as many socks in the basket, the time of execution of the second algorithm will only by m by n.
- The time of execution of the first algorithm will be instead m^2 by n.
- So if n=10, then using the second algorithm to match twice as many socks will take twice as much time.
- Instead, if n=10, then using the first algorithm to match twice as many socks will take four times as much time.