What is Computation?

Boolean Circuits and Finite State Machines

Carlotta Pavese

10.17.13

Outline

Review

Computation and Functionalism

- Functionalist understanding of minds: what they do

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Our Big Question

So what exactly is computation and are all mental activities just computation?

Today's Lecture

Part 1: Boolean Circuits

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6. Computers consist of Boolean Circuits, an FSM and another component we'll discuss next week

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  - The way they operate give rise to certain laws
  - These laws provide rules for correct reasoning

Boolean Logic

The Boolean Connectives and Laws of Thought

Negation ($\neg$): *Not*

A	$\neg$A
TRUE	FALSE
FALSE	TRUE

Conjunction ($\wedge$): *And*

A	B	A $\wedge$ B
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

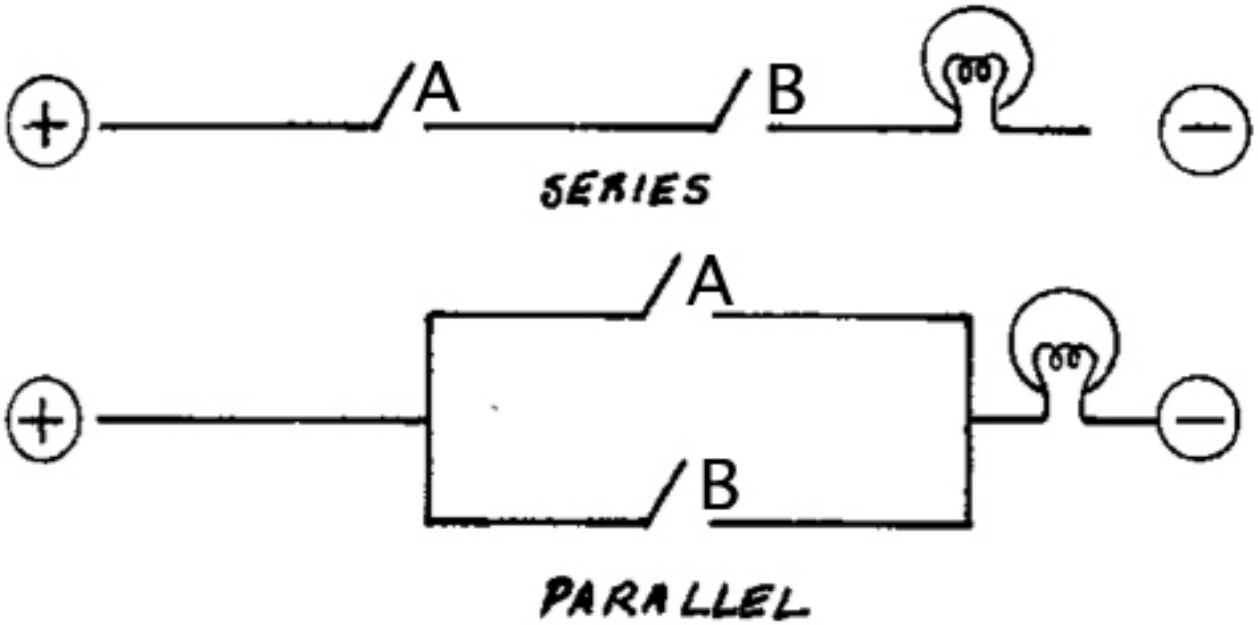
Disjunction ($\vee$): *Or*

A	B	A $\vee$ B
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

- $\neg$ flips truth/falsity
- $\wedge$ takes worstvalue
- $\vee$ takes bestvalue

Shannon's Insight

Electrical Boolean Circuits



1. Boole's variables become switches
2. Truth becomes current flowing (closed)
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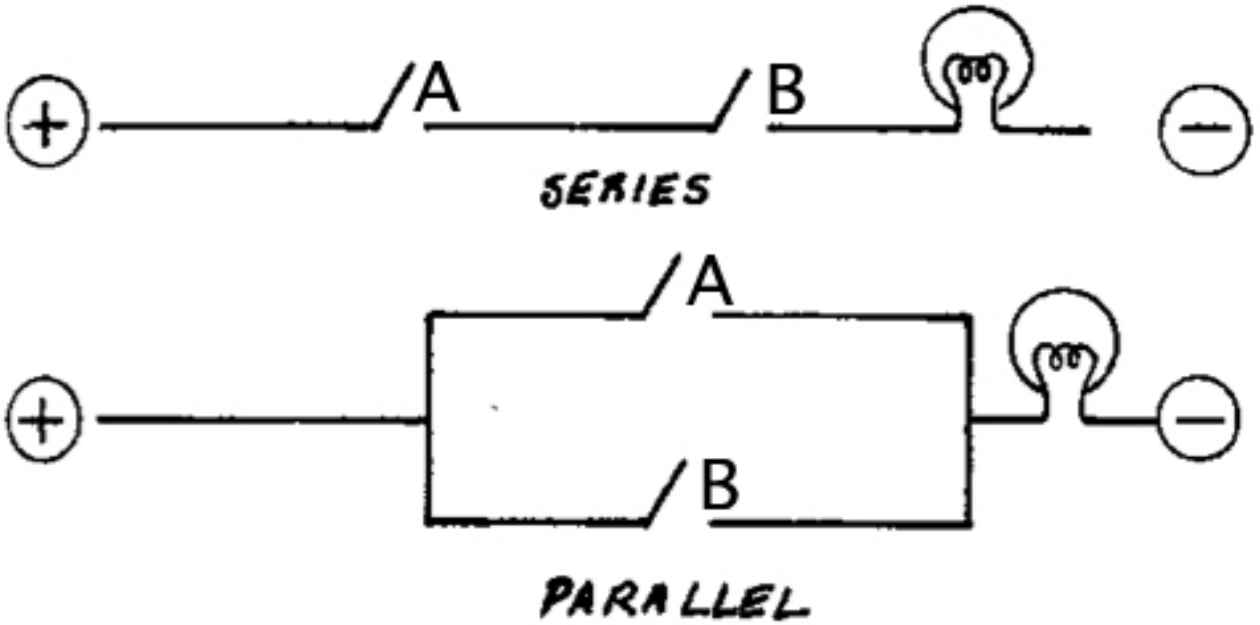

$A \wedge B$

$A \vee B$

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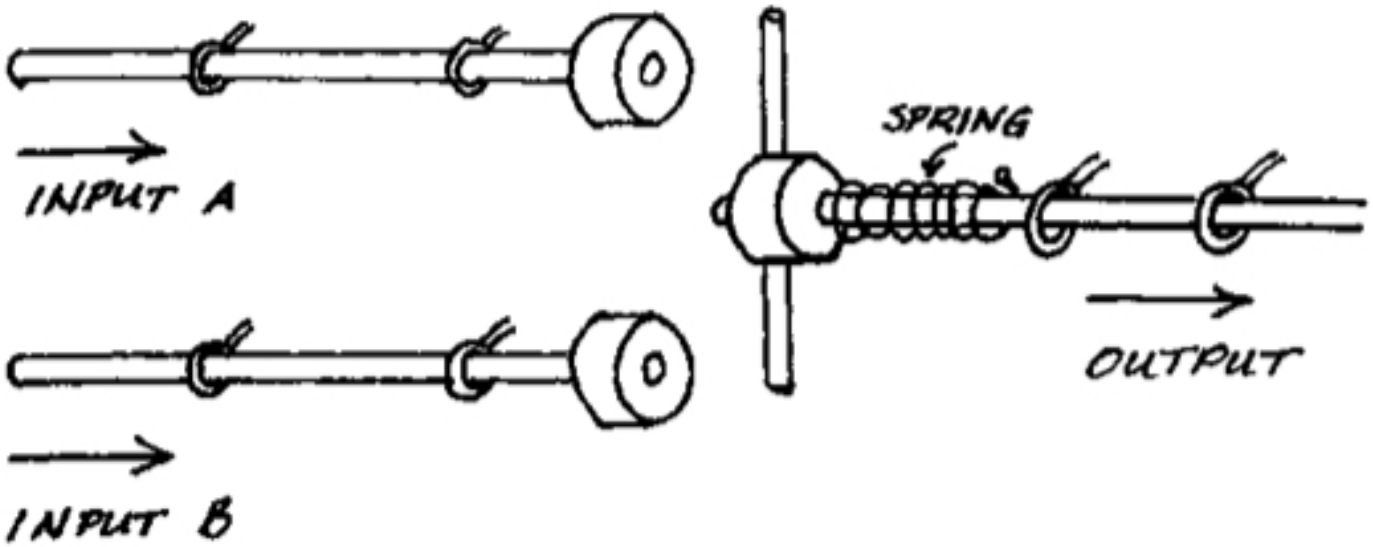
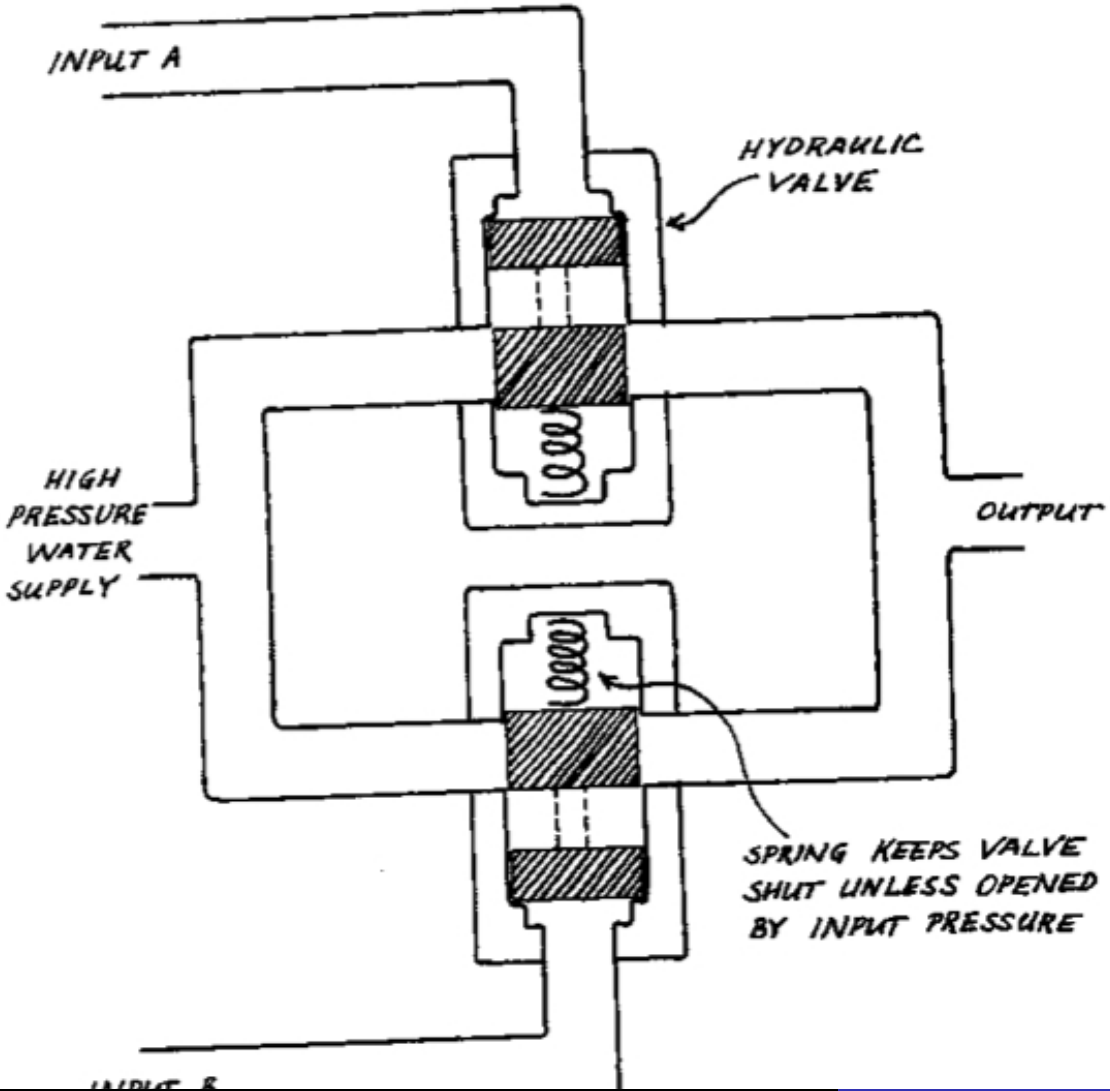
Mechanical OR Circuit



Shannon's Insight

Isn't Specific to Electricity: Hydraulic OR



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- **Definition**: A bit is any binary signal

Thinking of Bits as 1's and 0's

Forget About The Details, Example of OR

Disjunction and Mechanical OR

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FALSE	TRUE	TRUE
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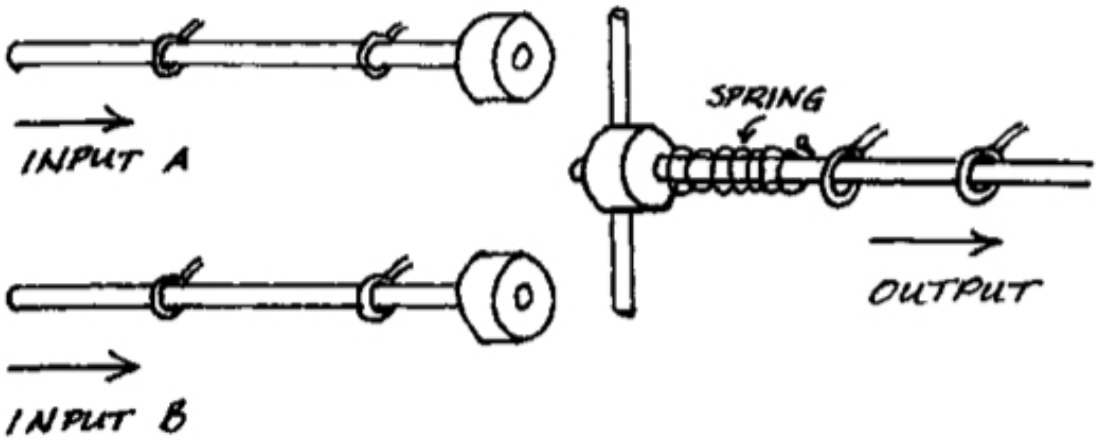

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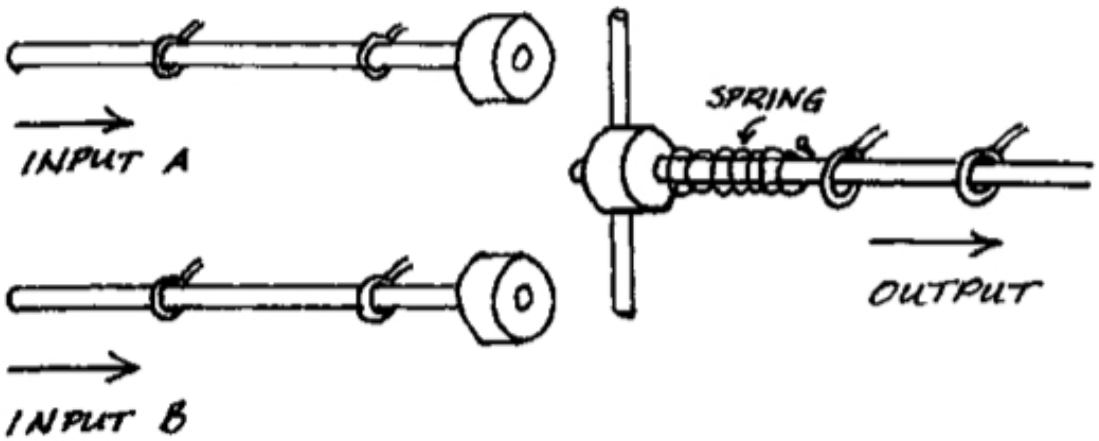

OR in Bits

A	B	A OR B
1	1	1
1	0	1
0	0	1
0	0	0

Forget what 1 & 0 really are and how those things are combined!

Logic Functions

Any Way of Crunching Bits

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2. These just take in 2 bits and spit out 1, how do we do tasks involving more inputs and outputs?
3. Why do we care about either of the questions above?

Answering the First Question

The Nature of Bit Crunching

Question 1

Are there any bit crunching tasks which (together) AND, OR and INVERT can't perform?

Answer

- No: together AND, OR and INVERT can perform *any* logic function

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- In fact: just OR + INVERT or AND + INVERT can perform any logic function
- Recall from last class that INVERT and OR can be combined to do what AND does

Answering the Second Question

More on the Nature of Bit Crunching

Question 2

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AND, OR and INVERT just take in 2 bits and spit out 1, how do we do tasks involving more inputs and outputs?

Answer

- By chaining together AND, OR and INVERT one can construct bit crunchers that take in more than 2 bits and spit out more than 1

Three Inputs

By Combining ORs

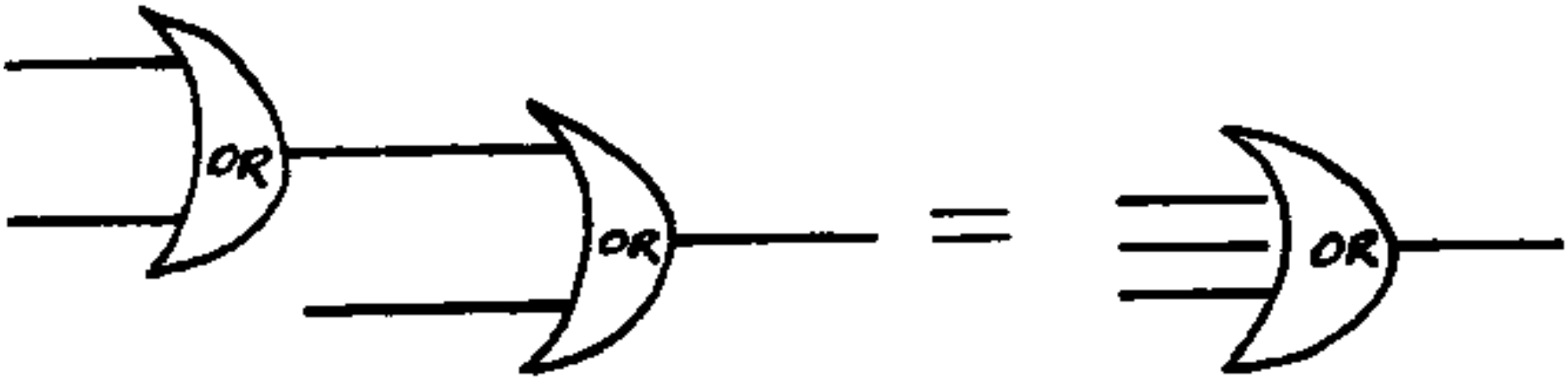

FIGURE 10

A three-input Or block made from a pair of two-input Or blocks

Majority Rules

Take-in 3, Put-out 1

Inputs			Majority Output
A	B	C	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
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Majority Rules Diagram, v1

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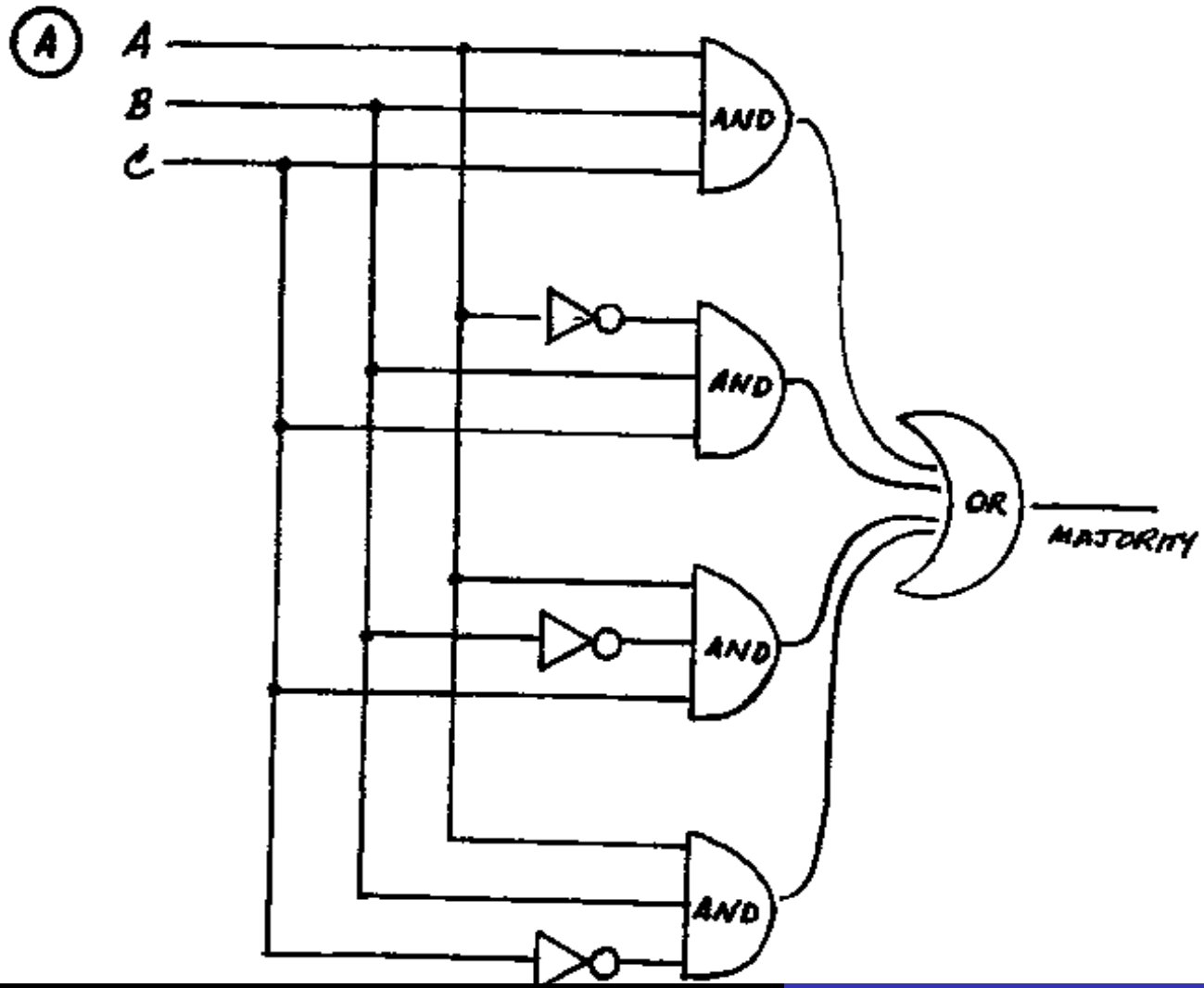

Majority Rules Diagram, v2

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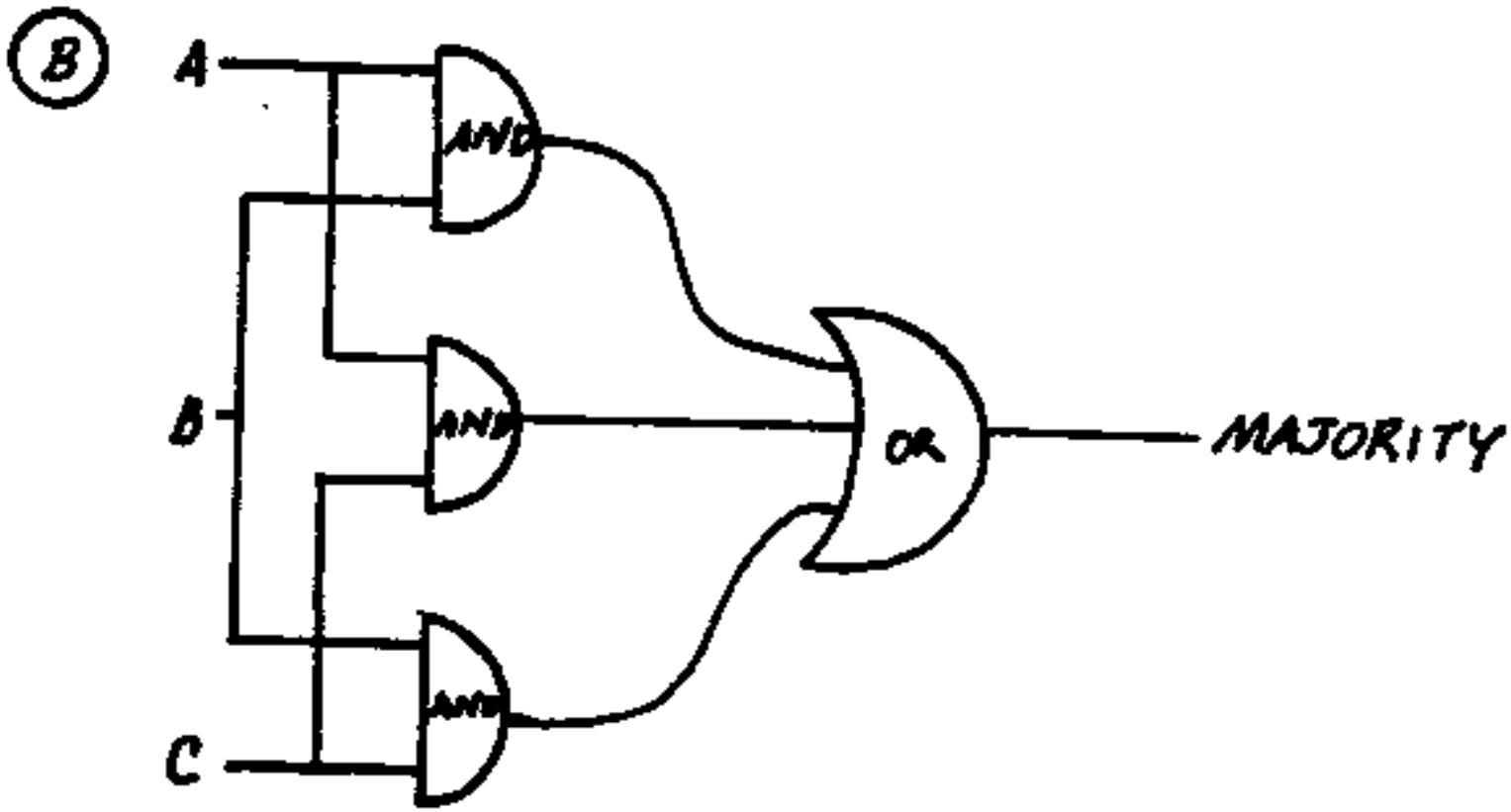

Answering Question 3

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Why care whether Boolean circuits can do any bit crunching task? Why care about more than 2 inputs or more than 1 output?

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- Now we know that any bit crunching task can be reduced to two incredibly simple tasks

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- Minds do things that require >2 inputs and outputs
  - E.g. judging the winner of Rock-Paper-Scissors

Rock-Paper-Scissors Judge

Ternary Input, Ternary Output!

Input A	Input B	Output
Scissors	Scissors	Tie
Scissors	Paper	A wins
Scissors	Rock	B wins
Paper	Scissors	B wins
Paper	Paper	Tie
Paper	Rock	A wins
Rock	Scissors	A wins
Rock	Paper	B wins
Rock	Rock	Tie

Rock-Paper-Scissors

Using Two Binary Inputs to Mimic One Ternary Input

Scissors = 01
Paper = 10
Rock = 11
A wins = 10
B wins = 01
Tie = 00

A Inputs	B Inputs	Outputs
01	01	00
01	10	10
01	11	01
10	01	01
10	10	00
10	11	10
11	01	10
11	10	01
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- Are minds just giant Boolean circuits? ('Logic blocks')

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- Machines which monitor and change an internal **register**: Finite State Machines
  - 'Register': a limited, finite memory
- Register keeps track of the previous input

Minds Aren't Just Logic Blocks

You Don't Always Do the Same Thing in Response to a Signal

- Logic blocks always react to the same signal/input in the same way, but things with minds don't!
- Minds react on the basis of their previous experiences!
- Even very simple machines do this:
  - Clicking pen, combo lock
- Machines which monitor and change an internal **register**: Finite State Machines
  - 'Register': a limited, finite memory
- Register keeps track of the previous input
- Register is itself is a Boolean circuit

Finite State Machines

Combo Lock For 0-5-2 (p.36)

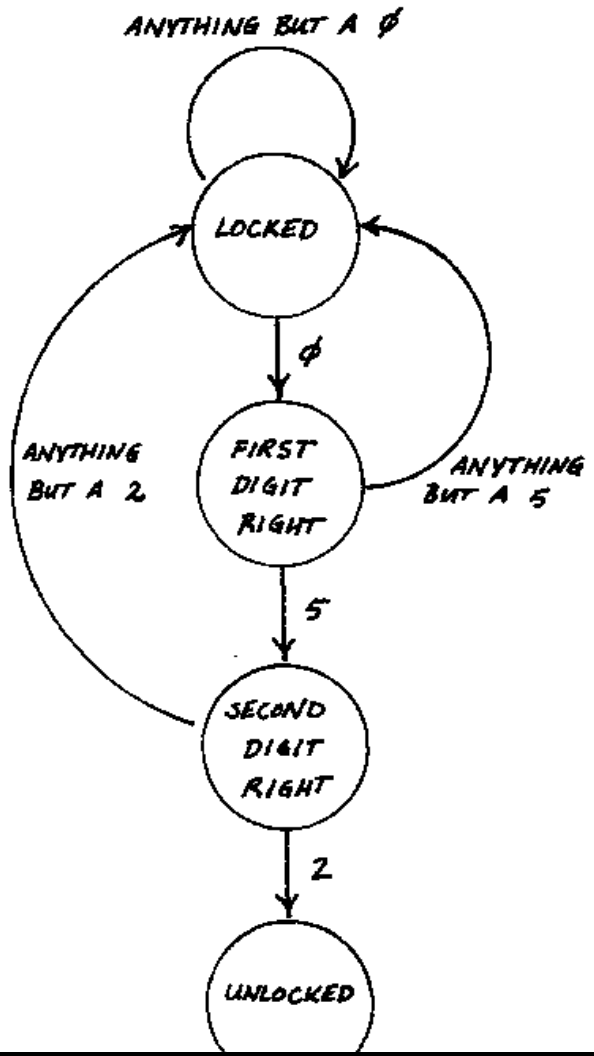
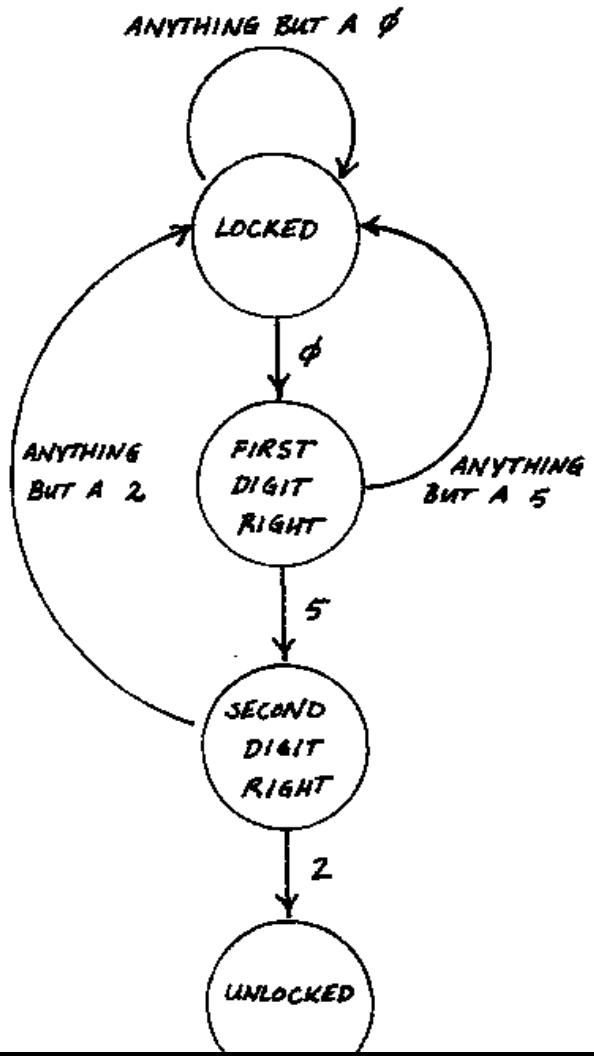


FIGURE 15

State diagram for a lock

Finite State Machines

Combo Lock For 0-5-2 (p.36)



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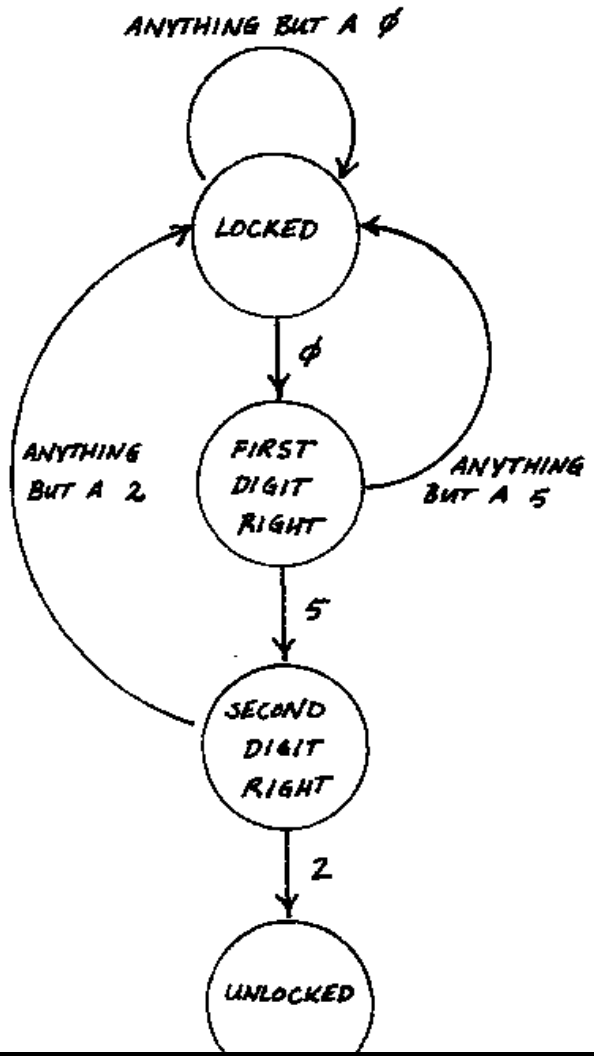

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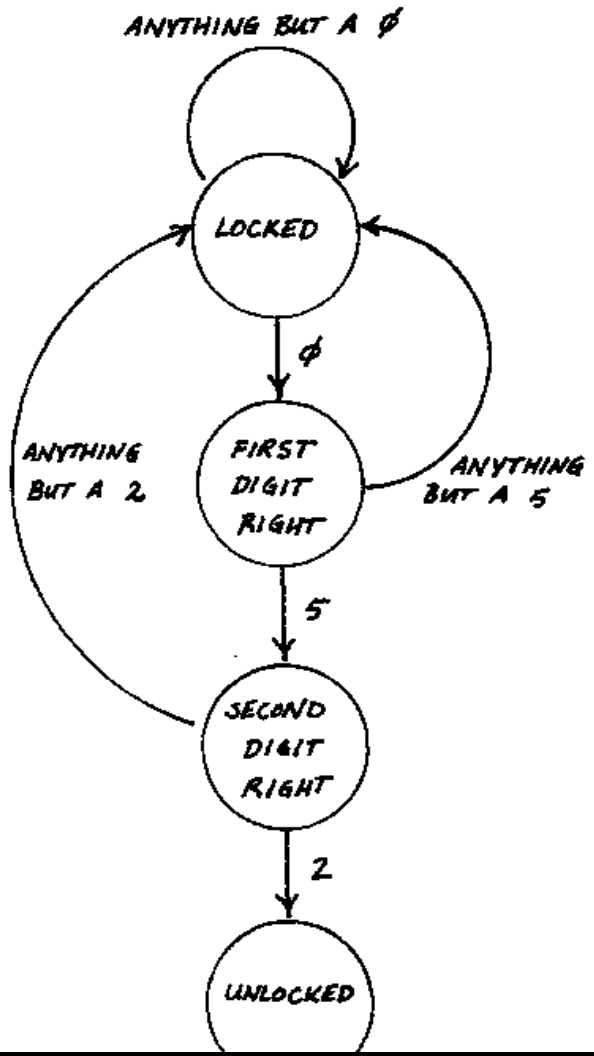

FIGURE 15

State diagram for a lock

FSMs Listen to Themselves

- Response = input + current state
- Current state = previous input

Stoplight Controller

As a Finite State Machine

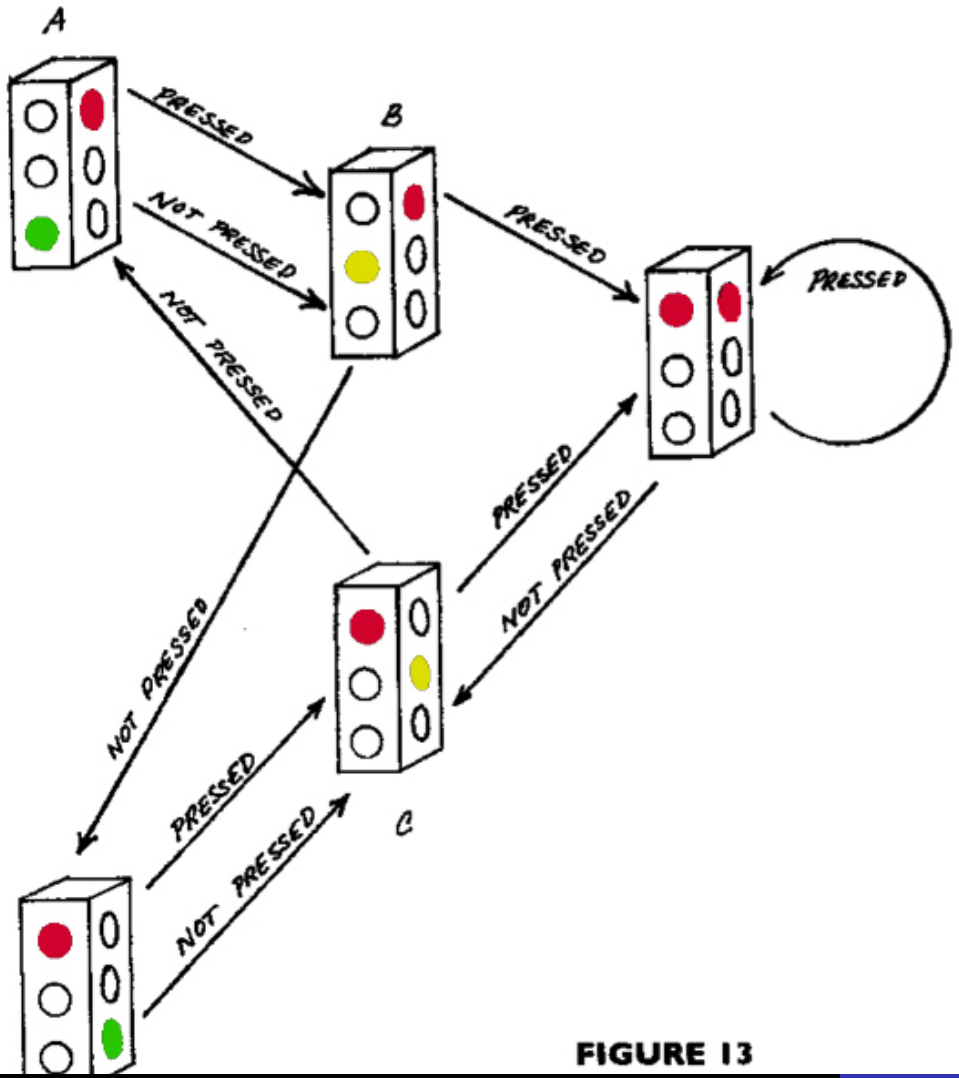

FIGURE 13

How A Stoplight Works

In Terms of Input/Output

Inputs:		*Outputs:*		
Walk Button	**Current State**	**Main Road**	**Cross Road**	**Next State**
Not Pressed	A	Red	Green	B
Not Pressed	B	Red	Yellow	D
Not Pressed	C	Yellow	Red	A
Not Pressed	D	Green	Red	C
Not Pressed	Walk	Walk	Walk	D
Pressed	A	Red	Green	B
Pressed	B	Red	Yellow	Walk
Pressed	C	Yellow	Red	Walk
Pressed	D	Green	Red	C
Pressed	Walk	Walk	Walk	Walk

Finite State Machines

The Mealy Machine

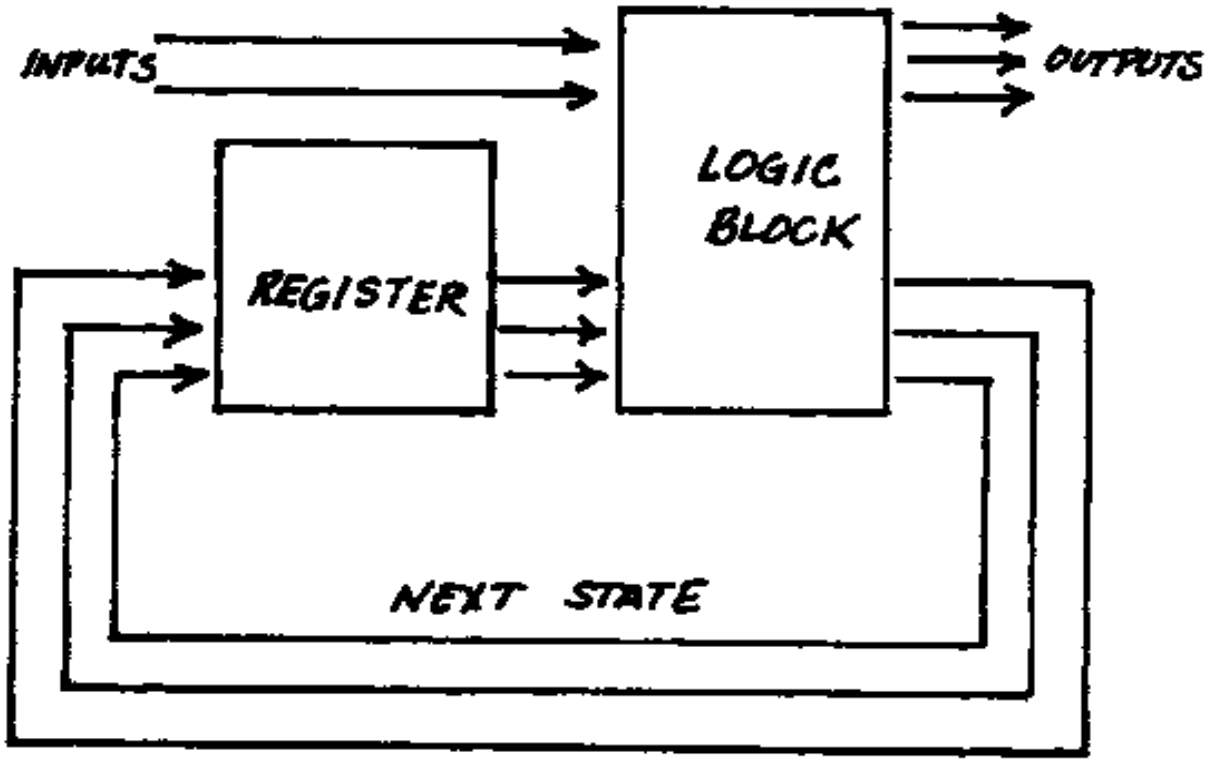

FIGURE 14

Finite-state machine, with logic block feeding register

(G.H. Mealy, "A Method for Synthesizing Sequential Circuits" 1955)

Finite State Machines

In General

In General

1. Register built from Boolean circuits

Finite State Machines

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- This allows finite state machines to recognize patterns

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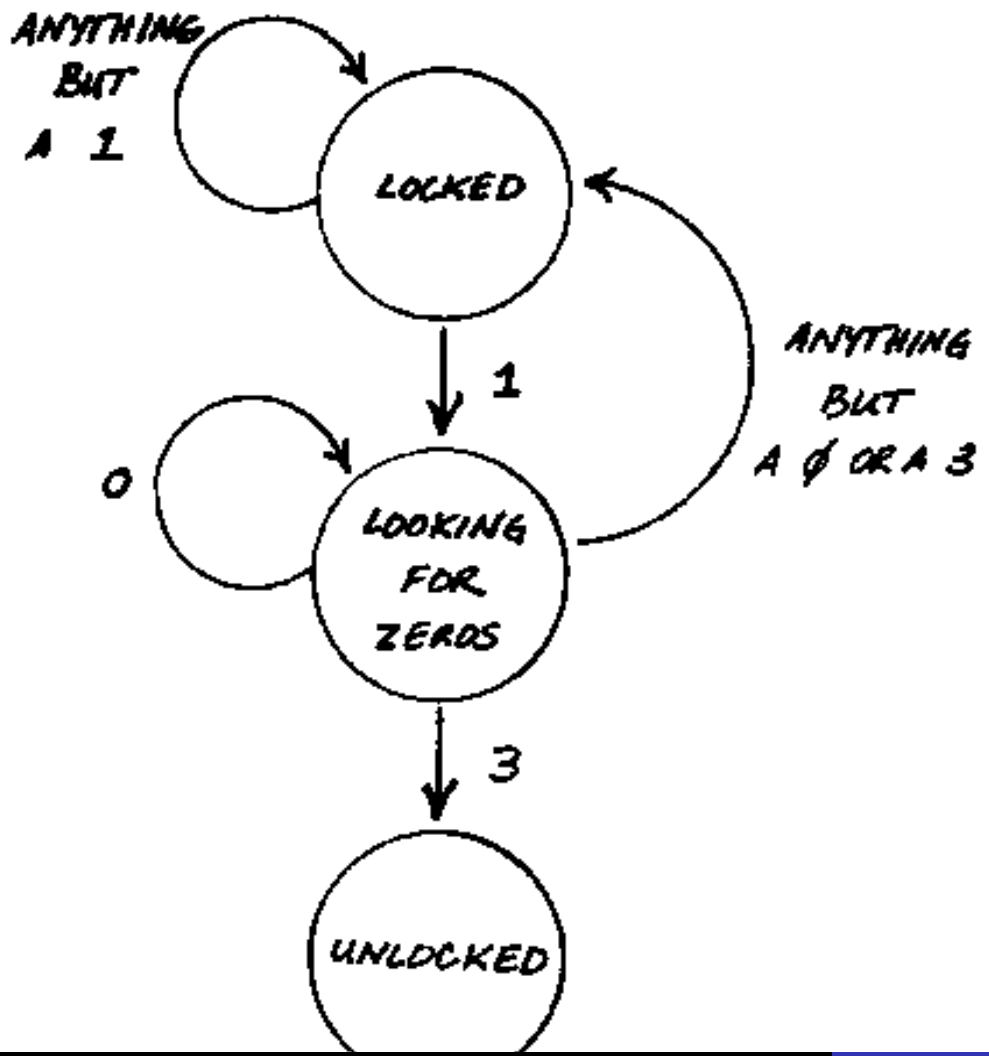
- This allows finite state machines to recognize patterns
- All patterns?

Finite State Machines

Recognizing Patterns: begins w/1, any number of 0s, ends w/3 (p.35)



Finite State Machines

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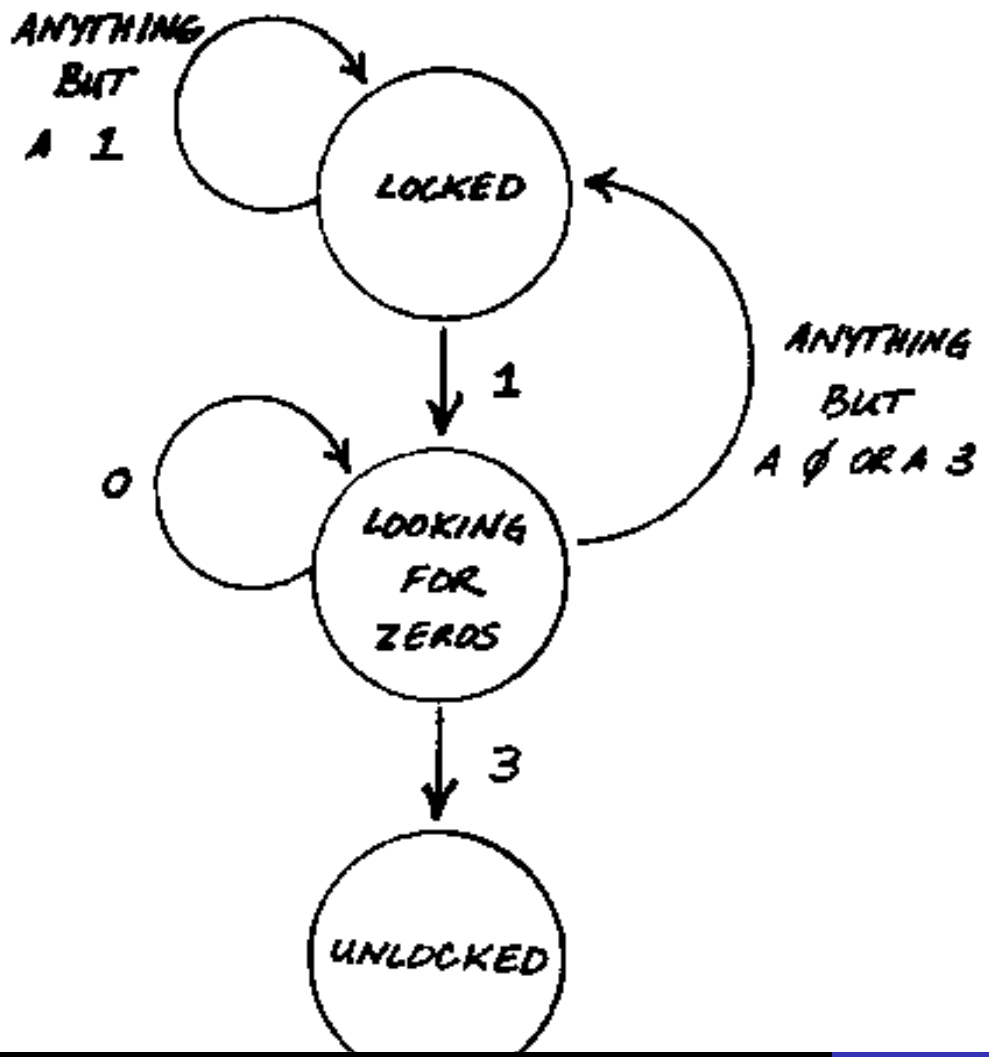

Important

There is one state for each digit that has to be remembered.

Finite State Machines

Recognizing Patterns

Palindrome

A string of symbols which is the same whether read backwards or forwards.

- E.g. 99, 757, 1001, abba, abccba, dad, mom

Finite State Machines

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Finite State Machines

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Why

To recognize a palindrome, you need to remember every digit of the first half. You need one state for each such memory. But there are infinitely many first halves, so you would need infinitely many states.

Finite State Machines

And Human Language

- Chomsky (*Syntactic Structures* 1957) argued an FSM could not recognize all sentences of a human language

Finite State Machines

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    (S_1 and S_2 are sentences)

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- But there are infinitely many!
- Think you've found the last sentence? Call it S.
- Isn't this a sentence: *I don't believe that S*?

Reading

For 10.22

Required

Ch. 3 of *The Pattern on the Stone*

Recommended