What is a Computer?

The Nuts and Bolts of Computation

Carlotta Pavese

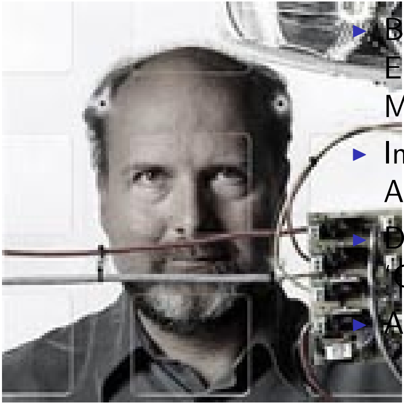
10.10

Outline

Outline

W. Daniel Hillis

Cast of Characters

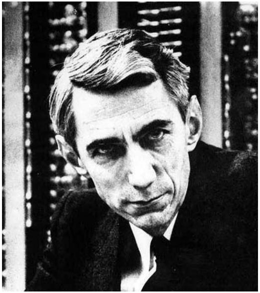


(American)

- BS in Math, MS, PhD in Electrical Engineering & Computer Science at MIT
- Important contemporary figure in Artificial Intelligence
- Designed parallel supercomputer 'Connection Machine'
- Author of *The Pattern on the Stone*

Claude Shannon

Cast of Characters



(American, 1916-2001)

- BS in Math & Electrical Engineering from University of Michigan
- MS in Electrical Engineering and PhD in Math at MIT
- Mathematician, electrical engineer & cryptographer
- The father of Information Theory

- Did foundational work on digital computers and digital circuits using work by George Boole

George Boole

Cast of Characters

(British, 1815-1864)

- Self-educated mathematician and philosopher
- Opened his own school at 19
- Professor at Queen's University (Ireland)
- In 1854 authored *An Investigation of the Laws of Thought*
- Discovered algebraic parallels between arithmetic and logic (Boolean Algebra)
- These parallels are one of the foundational insights that made the digital computer possible

Where are we?

The Mind-Body Question

The Mind-Body Question (Abstract, Philosophical)

- Is the mind part of the body?
- Is it separable? If so, how does it interact?

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Further Questions (Particular, Psychological)

- What are these things (thought, experience, emotion) that the mind does?
- How does it do them?

Where are we?

Trying to Answer the Mind-Body Question

Criterion 1

Connect mind and brain more closely than Dualism

- Illustrated by ‘interaction’ problem

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Criterion 3

Connect mind more closely than Behaviorism to internal psychology and working of the brain, without circularity

Functionalism

An Answer to the Mind-Body Question

Functionalism

1. The mind is what the brain does

Functionalism is Not Enough

Functionalism doesn't answer our further questions: what does the brain do, and how?

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1. The mind is what the brain does
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- Mind-body problem is addressed, Chauvinism avoided

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From Functionalism to Computation

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Computation as the Answer

Maybe brains, like your laptops, compute

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Computation as the Answer

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- But what is computation?
- How does it avoid Chauvinism? Can seeing, thinking, feel, etc. really arise from computation?

Outline

Boolean Logic

Introduction

An Investigation of the Laws of Thought, p.1

"The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed."

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The Basics

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  - There are rules for correct reasoning

Boolean Logic

The Boolean Connectives

Negation (¬): *Not*

A	¬A
TRUE	FALSE
FALSE	TRUE

Conjunction (∧): *And*

A	B	A ∧ B
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

Disjunction (∨): *Or*

A	B	A ∨ B
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

- ¬ flips truth/falsity
- ∧ takes worstvalue
- ∨ takes bestvalue

The Analogy With Arithmetic

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- $\neg(A \wedge B)$
- $\neg A \wedge B$
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Illustration

A	B	C	$(A \wedge B) \vee C$
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Remember Arguments?

Remember Validity and Soundness?

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An argument is valid just in case: the conclusion is true IF the premises are.

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- Good arguments = sound arguments

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Holy Matrimony

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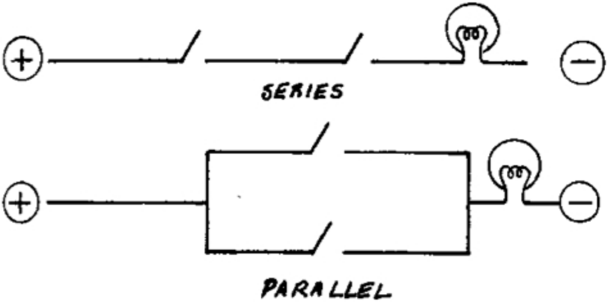
  - Just as algebra provides a Calculus for arithmetic
- These laws seem to provide a 'mechanical' method for reasoning correctly!

Outline

Shannon's Insight

Electrifying Boole



1. Variables are switches
2. Truth (1): letting current flow (closed)
3. Falsity (0): obstructing current (open)
4. Series=And, Parallel=Or

Shannon's Insight

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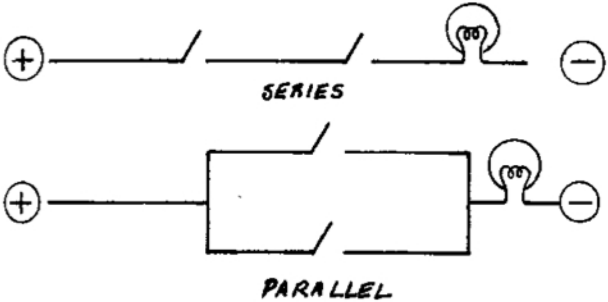
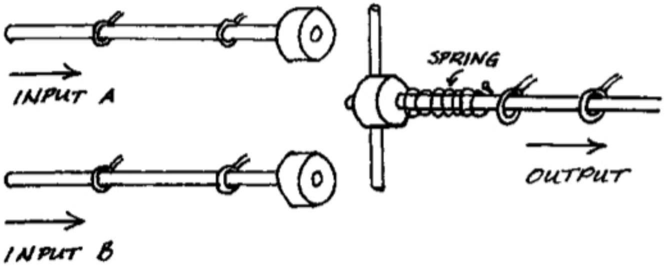


- There are two binary signals:
  1. The on/off current after switch 1
  2. The on/off current after switch 2
- *Lingo*: Bit=binary signal. A signal is a bit if and only if it can carry one of two different messages.

Shannon's Insight

Isn't Specific to Electricity: Mechanical OR

Mechanical OR:



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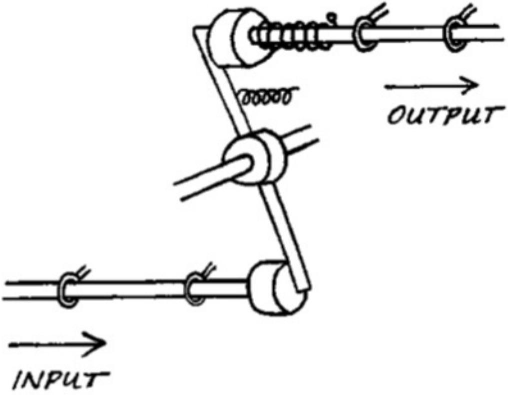

Shannon's Insight

Isn't Specific to Electricity: Mechanical AND Built from NOT and OR

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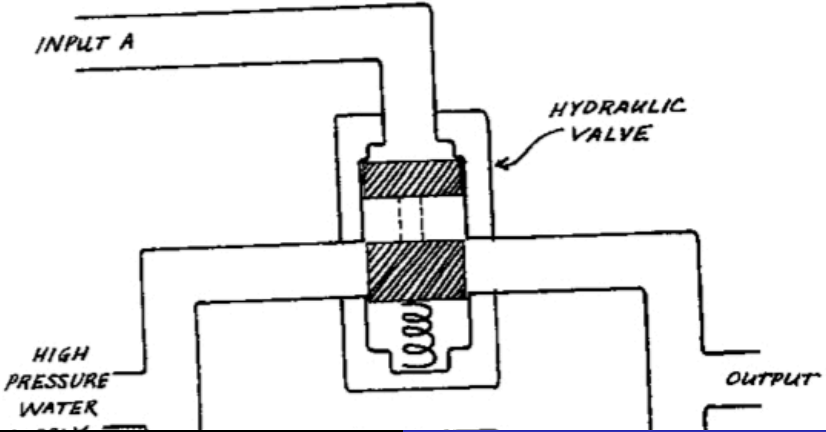

Shannon's Insight

Isn't Specific to Electricity: Hydraulic OR

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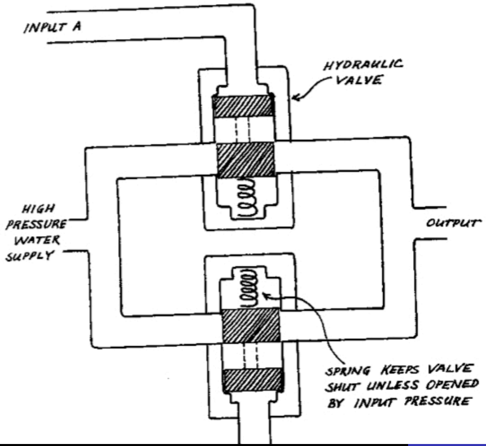

In Class Exercise

Becoming Bit Crunchers

The Exercise

By passing ‘bits’, and reacting like Boolean Circuits, arrange into a single circuit that corresponds to the following Boolean formula?

$(A \wedge B) \vee C.$

$(A \vee B) \wedge C.$

$(A \vee B) \wedge (C \wedge D)$

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- If we equate thinking with following these laws, we have a characterization of thought that is medium-independent: electric, mechanical, hydraulic
- No Chauvinsm, but perfectly physical!

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3. Are there any mechanisms that can't be built out of Boolean circuits?
4. Is a computer just a bunch of Boolean circuits?
5. How could a computer be flexible in what it does?
6. What exactly can a computer do?

Reading

For 10.17

- Ch.2 of *The Pattern on the Stone*